FROM: Encyclopedia of Physics: G.L. Trigg and R. Lerner, Eds. Addison-Wesley, NY 1980.

representation of $SU(2)$, or $SU(3)$ if hypercharge-changing currents are included; this implies that the commutator of a vector charge with an axial charge is again an axial charge. Then Gell-Mann postulated that the commutator of two axial charges be a vector charge, i.e., the algebra of all charges closes under commutation to the Lie algebra of $SU(2)\otimes SU(2)$, resp. $SU(3)\otimes SU(3)$. From this so-called *chiral symmetry* a large number of predictions were obtained, simply by taking matrix elements of the commutation relations between adequate states and using the Wigner–Eckart theorem again. Finally, going one step further, Gell-Mann postulated that the currents themselves satisfy a local $SU(3)\otimes SU(3)$ algebra: this is the famous *current algebra*. Extensions like $SU(6)\otimes SU(6)$ have been proposed, but the idea is the same. These developments represent a remarkable evolution since the early applications of group theory. The precise structure of the various hadronic currents is unknown, but only their symmetry properties are important: the line of thought is exactly opposite to the one originally used, e.g. in atomic physics!

NEW DEVELOPMENTS

Recent developments in particle physics have been spectacular, and more than ever they rely heavily on group-theoretical ideas. First, when $SU(3)$ was introduced, it was soon suggested by Gell-Mann that all known hadrons could be thought of as bound states of three elementary buildings blocks, the so-called *quarks*, and their antiparticles. This model, naive at first but steadily refined, has been remarkably successful for describing the dynamical properties of hadrons (although the quarks themselves have never been found). Recently, however, the discovery of totally new particles has forced the theorists to enlarge their model. First, in 1974–1975, came the charmonium family ($\psi,\psi',\psi''\ldots$), whose properties can be best understood by the existence of a new quantum number called *charm*; this demands a fourth quark. Similarly, the upsilon family, discovered in 1977, requires a fifth quark; and so the whole game must be played once again, replacing $SU(3)$ first by $SU(4)$, then by $SU(5)$.

By far the most promising development, however, is the emergence of the so-called *gauge theories*, which are based on an extension of the concept of symmetry. Within a field theory, an internal symmetry is said to be *global* if the action of the symmetry group on the field $\varphi(x)$ is independent of the space-time point x; this is the concept used so far. The symmetry is called *local* if the action is, in addition, allowed to vary from point to point; such a theory is called a gauge field theory, and the local symmetry group is called a gauge group. The idea goes back to H. Weyl in 1918, who treated electromagnetism as a gauge theory based on the commutative gauge group $U(1)$. A noncommutative theory, based on $SU(2)$, was proposed by C. N. Yang and R. L. Mills in 1954, but the gauge concept did not become popular until the young Dutch physicist G. 't Hooft proved in 1971 that a noncommutative gauge field theory is renormalizable (i.e., susceptible of giving consistently finite predictions). The key point in his proof was that a clever use of group identities produced sufficiently many cancellations between potentially divergent terms.

Since then, gauge theories (and with them, differential geometry) have invaded the whole field of particle physics, with rather remarkable results. On the one hand, the model of S. Weinberg and A. Salam, based on the gauge group $SU(2)\otimes U(1)$ (which, incidentally, requires a sixth quark), gives a unified description of weak and electromagnetic interactions; it has accumulated excellent experimental support and is fast acquiring the status of an established theory (although a key ingredient—particles known as Higgs bosons—has not been found so far). On the other hand, a new theory of strong interaction, called *quantum chromodynamics* (QCD) has emerged; it is also a gauge theory, with gauge group $SU(3)$, that generalizes the old quark model by assuming that every quark comes in three types (or *colors*). If we add that general relativity can also be viewed as a gauge theory, the fruitfulness of the gauge idea, based in an essential way on group-theoretical techniques, is impressive indeed!

CONCLUSION

It seems fair to say that group theory has grown into one of the essential tools of contemporary physics. Besides its fundamental role in relativity, it has provided physicists with a remarkable analyzing power for exploiting known symmetries, and thereby with a considerable predictive capability, precisely in cases where the basic physical laws are unknown. One of the striking aspects is its versatility: going from the rather restrictive study of exact symmetries to that of approximate ones, more and more badly broken, group theory has pervaded all fields of physics, often in a fundamental way. Except for calculus and linear algebra, no mathematical technique has been so successful.

See also CONSERVATION LAWS; CRYSTAL SYMMETRY; ELEMENTARY PARTICLES IN PHYSICS; GAUGE THEORIES; INVARIANCE PRINCIPLES; ISOSPIN; LIE GROUPS; OPERATORS; RELATIVITY, SPECIAL; $SU(3)$ AND HIGHER SYMMETRIES.

BIBLIOGRAPHY

H. Weyl, *The Theory of Groups and Quantum Mechanics* Dover, New York, 1950 (original, 1st German ed., 1928).

E. P. Wigner, *Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra*. Academic Press, New York. 1959 (original, 1st German ed., 1931).

F. J. Dyson, *Symmetry Groups in Nuclear and Particle Physics* (a lecture note and reprint volume). Benjamin, New York. 1966.

E. M. Loebl, ed., *Group Theory and Its Applications*, Vols. 1–3. Academic Press, New York, 1968–1975.

Gyromagnetic Ratio

Jack H. Freed

The gyromagnetic ratio (or magnetogyric) ratio γ is defined as the ratio of the magnetic moment μ to the angular momentum **J** for any system. Specifically, one introduces for electrons with electron spin angular momentum $\mathbf{J} = \hbar\mathbf{S}$ (where $\hbar = h/2\pi$ and h is Planck's constant) the electron gyromagnetic ratio γ_e by $\boldsymbol{\mu}_e = -\gamma_e\hbar\mathbf{S}$, where the negative sign represents the fact that the spin and moment are oppositely directed. For a nucleus, which is a composite particle with total spin **I**, one introduces the nuclear gyromagnetic ratio γ_I by $\boldsymbol{\mu}_I = \gamma_I\hbar\mathbf{I}$. Both $\boldsymbol{\mu}_I$ and **I** (or $\boldsymbol{\mu}_e$ and **S**) must be regarded as quantum-mechanical operators. In the case of nuclear moments, the defining equation for γ_I must be considered as implying that the expectation values of $\boldsymbol{\mu}_I$ and **I** are taken for the given state (usually the ground state) of the nucleus. It then follows from symmetry considerations expressed in the Wigner–Eckart theorem that $\boldsymbol{\mu}_I$ and **I** may be taken as collinear, with $\gamma_I\hbar$ the proportionality constant.

A classical spinning spherical particle with mass m and charge e can be shown to give rise to a magnetic moment $e\hbar/2mc$, where c is the velocity of light. For an electron, this moment is known as the Bohr magneton, $\beta = 0.9274096 \times 10^{-20}$ erg/G. But one has $\gamma_e\hbar = g_s\beta$, where g_s is the anomalous g value of the electron spin, which was first derived from the relativistic Dirac equation to be exactly 2. Schwinger showed how to correct this for quantum-electrodynamic effects to first order in $\alpha = e^2/\hbar c$ to give $g_s = 2(1 + \alpha/2\pi) = 2.0023$. The most accurate theoretical calculation, due to Sommerfeld, gives $g_s = 2.0023192768$, which is in excellent agreement with the value of 2.002319244, obtained by Wilkinson and Crane by electron-beam experiments. Earlier atomic-beam measurements by Kusch on the hydrogen atom, for which corrections must be made for the relativistic mass change due to binding, yielded $g_s = 2.002292$.

The nuclear gyromagnetic ratio, representing a composite nuclear property, may be measured very accurately by molecular-beam techniques. Also useful are nuclear magnetic resonance and optical, microwave, electron paramagnetic, Mössbauer, and electric quadrupole resonances. One typically introduces the nuclear magneton $\beta_N = 0.5050951 \times 10^{-23}$ erg/G and the nuclear g value; thus $\gamma_I\hbar = g_I\beta_N$. One finds for the proton $g_P = 5.585564$ with $\gamma_P = 2.6751965 \times 10^4$ rad/s G (corrected for diamagnetism of H_2O).

See also MAGNETIC MOMENTS; NUCLEAR MOMENTS.

BIBLIOGRAPHY

H. Kopfermann, *Nuclear Moments*. Academic Press, New York 1958.

C. M. Lederer, J. M. Hollander, and I. Perlman, *Tables of Isotopes*. Wiley, New York, 1967.

N. F. Ramsey, *Molecular Beams*. Oxford, London, 1956.

B. N. Taylor, W. H. Parker, and D. N. Langenberg, *Rev. Mod. Phys*. 41, 375 (1969).