

# Supplementary Information

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### A. NUMBER OF FREQUENCY GRID POINTS NEEDED FOR AN ACCURATE 2D-ELDOR SPECTRUM

To estimate the number of grid points necessary to obtain an accurate 2D-ELDOR spectrum, we first calculate a reference 2D-ELDOR spectrum with  $n = 1000$  frequency grid points through the regular Arnoldi method. We can be confident that  $n = 1000$  corresponds to an extremely high resolution 2D-ELDOR spectrum. Then for  $n = 100, 200, 300, 500$ , we calculate the 2D-ELDOR spectrum through the regular Arnoldi method again. For each value of  $n$ , we use spline interpolation to interpolate to the reference case ( $n = 1000$ ). For each point in the 2D-frequency grid of  $1000 \times 1000$  points, we then evaluate the relative error between the true value of  $S_{c-}(f_1, f_2)$  and the interpolated value of  $S_{c-}(f_1, f_2)$  based on the smaller value of  $n$ . The maximum of the relative errors (MRE) over the  $1000 \times 1000$  grid, for each value of  $n$ , is shown in Fig. A.1. As expected, this error decays to 0 as the grid becomes finer, i.e.,  $n$  increases. We take our threshold to be 1% MRE, to guarantee a sufficiently accurate calculation for purposes of non-linear least squares fitting to experimental data.

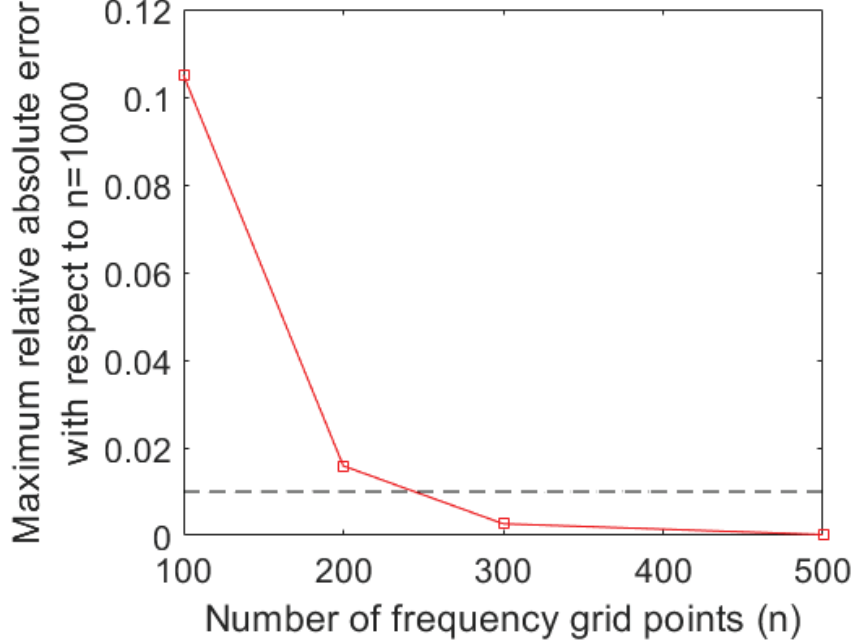


FIG. A.1:  $R_{\parallel} = 10^5 s^{-1}$ ,  $k_{sym} = 10^6 s^{-1}$ , mixing time  $T_{mix} = 500 \text{ ns}$ . The black dotted line indicates the 1 % error level, corresponding to  $n \sim 250$ . Other parameters are as in Fig. 6.

## B. NUMBER OF FREQUENCY GRID POINTS NEEDED FOR AN ACCURATE CW SPECTRUM

To estimate the number of grid points necessary to obtain an accurate CW spectrum, we first calculate a reference integrated CW spectrum with  $n = 1000$  frequency grid points. We can be confident that  $n = 1000$  corresponds to an extremely high resolution CW spectrum. Then for  $n = 100, 200, 300, 500$ , we calculate the integrated CW spectrum again. For each value of  $n$ , we use spline interpolation to interpolate to the reference case ( $n = 1000$ ). For each point in the 1D-frequency grid of 1000 points, we then evaluate the relative error between the true value of  $S_{c-}(f_1, f_2)$  and the interpolated value of  $S_{c-}(f_1, f_2)$  based on the smaller value of  $n$ . The maximum of the relative errors (MRE) over the 1000 frequency grid points, for each value of  $n$ , is shown in Fig. B.1. As expected, this error decays to 0 as the grid becomes finer, i.e.,  $n$  increases. We take our threshold to be 1% MRE, to guarantee a sufficiently accurate calculation for purposes of non-linear least squares fitting to experimental data.

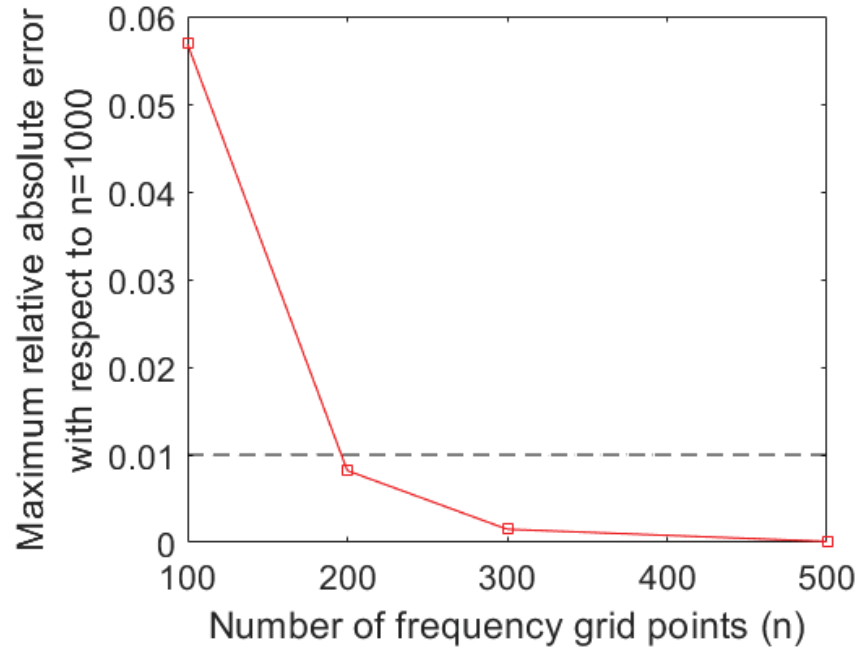


FIG. B.1:  $R_{\parallel} = 10^5 s^{-1}$ , diffusion tilt angle  $\beta_d = 90^\circ$ . The black dotted line indicates the 1% error level, corresponding to  $n \sim 200$ .