

# A two-dimensional Fourier transform electron-spin resonance (ESR) study of nuclear modulation and spin relaxation in irradiated malonic acid

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Nuclear modulation in electron-spin-echo spectroscopy is conventionally studied by one-dimensional electron-spin-echo envelope modulation (1D-ESEEM). Two-dimensional Fourier transform electron-spin resonance (2D-FTESR) studies of nuclear modulation have the promise of enhancing the spectral resolution and clarifying the key details of the relaxation processes. We present a 2D-FTESR study on single proton nuclear modulation from  $\gamma$ -irradiated malonic acid single crystals to test the validity of the Gamliel-Freed theory and to assess the value of the new methods. The two pulse spin-echo correlation spectroscopy (SECSY) spectra as a function of orientation of the single crystal show very good agreement with the Gamliel-Freed theory extended to the general case of nonaxially symmetric hyperfine interaction. It is very simply affected by spin relaxation, such that relative intensities are essentially unaffected. Thus SECSY-ESR can most reliably be utilized for studying nuclear modulation. Stimulated SECSY provides the simplest nuclear modulation patterns, which, however, do exhibit the suppression effect well known in three-pulse ESEEM studies. Two-dimensional electron-electron double resonance (2D-ELDOR) provides nuclear modulation patterns similar to that of SECSY-ESR, so the suppression effect is absent. Both three-pulse methods exhibit complex relaxation behavior which can affect relative intensities. This is a feature characteristic of three-pulse ESEEM, but is not well understood. It is shown how the 2D-FTESR methods enable one to obtain the details of the complex spin relaxation, and in the process, obtain very good agreement between experiment and theory. 2D-ELDOR exhibits exchange cross peaks as well as coherence peaks from the nuclear modulation. It is shown how experiments, as a function of mixing time, enable one to separate the effects of the two. It is pointed out that such experiments are in the spirit of 3D spectroscopy. A new observation of the broadening of the 2D-ELDOR main peaks with an increase in mixing time is ascribed to the effects of solid-state dynamical processes that are slow on the ESR time scale and may thus be studied in "real time" in such experiments. The analysis of spin relaxation in this study is enabled by a full Liouville space derivation of the combined effects of nuclear modulation and spin relaxation in 2D-FTESR.

## I. INTRODUCTION

Nuclear modulation of the electron-spin-echo decay was first reported in 1961<sup>1,2</sup> and thereafter was developed both theoretically and experimentally in the context of electron-spin-echo envelope modulation (ESEEM).<sup>3,4</sup> ESEEM has been studied over the ensuing years with the objective of obtaining structural information near paramagnetic centers in solid and glassy matrices.<sup>5-10</sup> This method has been most successful in dealing with nuclei that are weakly coupled to the electron spin by dipolar interactions. In these cases, the depth of the ESEEM can be exploited to estimate distances between the nuclei and the unpaired electron. Given that the ESEEM usually observed is due to weakly interacting nuclei, the structural information that is obtained is typically complementary to that obtained from electron-nuclear double resonance (ENDOR).<sup>6,11-14</sup>

An important recent development in electron-spin resonance (ESR) has been Fourier transform (FT) and two-dimensional FT-ESR (2D-FT-ESR).<sup>15-22</sup> These developments have the technical advantages of (a) greatly reduced

data acquisition times because the whole spectrum is irradiated simultaneously (i.e., the multiplex advantage) with the use of very narrow nonselective pulses (widths  $\sim 5$  ns) which are significantly shorter than the relevant  $T_2$ 's, thereby yielding (b) reduced dead time and spectral distortions arising from the use of long pulses. More important is the availability of a whole class of new experiments to study both motions and structure. In this paper, we address the class of 2D-FT-ESR experiments for the study of nuclear modulation.

In a recent theoretical study by Gamliel and Freed (GF),<sup>23</sup> it was shown that 2D-FT-ESR methods should, in general, provide greater spectral resolution of nuclear modulation with the availability of the second spectral dimension. In fact, the first demonstration of the value of a second spectral dimension in the study of ESEEM was provided by Merks and de Beer.<sup>24</sup> They utilized a standard ESEEM technique based on the three-pulse stimulated echo, which measures just the echo maximum as a function of two different pulse delay times [ $T$  and  $t_1$  in Fig. 1(d)]. Also notable was the two-pulse ESEEM experiment of Isoya *et al.*<sup>25</sup> who measured the echo maximum as a func-

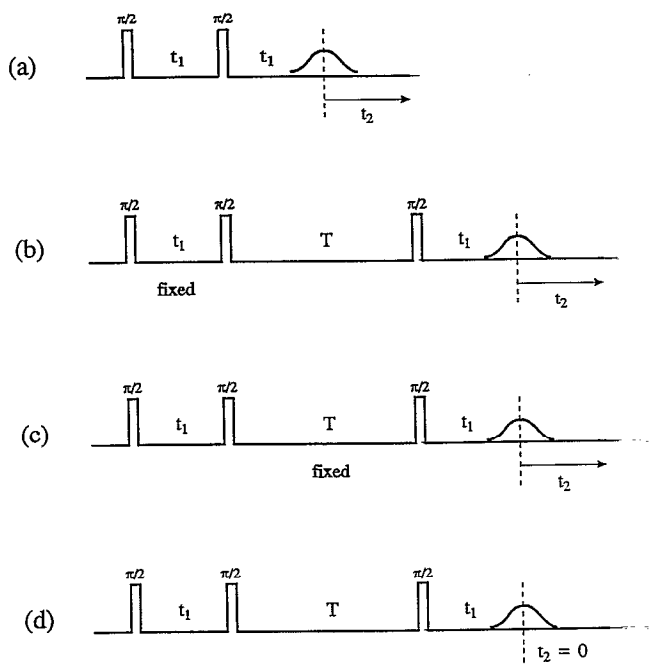


FIG. 1. The 2D-FTESR pulse sequences for echo signals: (a) SECSY—one varies  $t_1$  and  $t_2$ ; (b) stimulated SECSY—one varies  $T$  and  $t_2$  with fixed  $t_1$ ; (c) echo-ELDOR—one varies  $t_1$  and  $t_2$  with fixed mixing time  $T$ ; (d) Merks-de Beer 2D-ESEEM experiment—one varies  $t_1$  and  $T$  and monitors the echo height.

tion of  $t_1$  [cf. Fig. 1(a)] and repeated the experiment for different values of the d.c. magnetic field  $B_0$  to obtain a two-dimensional representation. These techniques are based on selective excitation and/or detection and do not have the multiplex advantage of a technique based on modern FT methodology. We distinguish the methods described in this paper by their full utilization of nonselective irradiation and their use of modern FT methods.

Patyal *et al.*<sup>26,27</sup> have recently demonstrated the capability of performing 2D-FT-ESR on spectra in the rigid and near rigid limits in a disordered matrix. They employed the GF theory to interpret the 2D nuclear modulation observed from a nitroxide in a glycerol/H<sub>2</sub>O host and showed the sensitivity of the 2D-FT-ESR methods to structural detail. It is known that rotational motions significantly affect nuclear modulation,<sup>19,20,26–28</sup> and Patyal *et al.* were able to illustrate these effects in their 2D-FT-ESR. In general, relaxation mechanisms can affect the observed nuclear modulations in terms of intensities and resolution, a matter that is still not well understood.

In this paper, we present a study of 2D-FT-ESR experiments on the nuclear modulation from a  $\gamma$ -irradiated single crystal of malonic acid (Fig. 2). This is meant as a prototype study to carefully demonstrate and analyze the capabilities of 2D-FT-ESR experiments in the study of nuclear modulation. The reasons for choosing the malonic acid radical are that (i) it is a simple  $S=1/2$ ,  $I=1/2$  spin system, which is easy to analyze; and (ii) detailed structural information is available by x-ray crystallography<sup>29</sup> and from previous ESR studies.<sup>30,31</sup> We are able to study

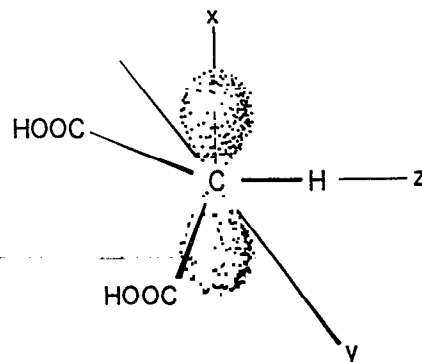


FIG. 2. Relation of the structure of the malonic acid radical  $\cdot\text{CH}(\text{COOH})_2$  to the principal axes ( $x,y,z$ ) of the hyperfine tensor. In this orthogonal system of coordinates, the  $z$  axis is along the CH bond direction and the  $x$  axis is perpendicular to the plane of the three carbon atoms (Ref. 30). The direction of the external static field  $H_0$  is defined by the angles  $\theta$  and  $\phi$ , where  $\theta$  is the angle between  $H_0$  and the  $z$  axis, and  $\phi$  is the angle between the  $x$  axis and the projection of  $H_0$  on the  $xy$  plane.

the various 2D-FT-ESR spectra as a function of sample orientation, which leads to dramatic variation of the observed nuclear modulation in a way that provides an excellent test of the theory. We are able to report on the generally excellent agreement obtained with GF theory after generalizing it to include nonaxially symmetric hyperfine (and  $g$ ) tensors.

This work also illustrates another virtue of our 2D-FT-ESR methods with short nonselective pulses, viz., it is possible to obtain good nuclear modulation from strongly coupled nuclei such as is the case for the  $\alpha$  proton in malonic acid. In order to perform accurate studies on nuclear modulation, it is necessary to excite the allowed and forbidden ESR transitions simultaneously. Their difference in frequency depends on both the hyperfine (hf) tensor and the nuclear Larmor frequency  $\omega_n$ . In general, it is difficult to adequately excite nuclear modulation of protons because  $\omega_n$  is large, but Patyal *et al.* showed that 5 ns pulses were adequate to accomplish this in the case of relatively weakly coupled protons,<sup>26,27</sup> and in the present work, we find that this is so for a strongly coupled proton where all principal components of the hf tensor are greater than  $\omega_n$ . Thus 2D-FT-ESR can deal effectively with weakly and strongly coupled nuclei including protons.

In this paper, we consider three canonical 2D-FT-ESR experiments that are based on echo decays. The first is the two-pulse spin-echo correlation spectroscopy (SECSY)-ESR experiment in which one collects the full echo decay as one steps out the time  $t_1$  between the two pulses [cf. Fig. 1(a)] followed by a double FT.<sup>23,26</sup> This technique leads to coherence cross peaks due to the nuclear modulation. These coherence cross peaks are analogous to those observed in COSY (correlation spectroscopy) and SECSY NMR arising from  $jj$  coupling,<sup>32</sup> although the formulation of nuclear modulation in ESR is more complex. This is the basic experiment which generalizes the standard two-pulse ESEEM experiment to two dimensions. There are two

three-pulse 2D-FT-ESR experiments which are based on the stimulated echo—stimulated SECSY [cf. Fig. 1(b)] and 2D-electron—electron double resonance (ELDOR)-echoes [cf. Fig. 1(c)]. Since there are actually three different times here, viz.  $t_1$  the preparation time,  $T$  the mixing time, and  $t_2$  the acquisition time, this is, in principle a 3D experiment.<sup>33</sup> The 2D experiments—stimulated SECSY (in which  $T$  and  $t_2$  are “varied”) and 2D-ELDOR (in which  $t_1$  and  $t_2$  are varied) are two of the three possible 2D projections of the full 3D experiment. The third is, of course, the Merks and de Beer experiment ( $t_1$  and  $T$  is varied), but it is the one that lacks the multiplex advantages referred to above. One finds that echo-ELDOR for short mixing times,  $T$  yields results very similar to SECSY-ESR for the structure of the coherence cross-peaks due to nuclear modulation. As  $T$  increases, cross peaks due to exchange processes can be observed to grow in. We consider, in our study, how coherence cross peaks and exchange cross peaks can be distinguished by such experiments. In general, stimulated SECSY and 2D-ELDOR are more sensitive to the details of spin relaxation than is SECSY. In the original GF theory, relaxation was treated in a very simple manner in order to avoid the full Liouville space analysis required for a relaxation-matrix<sup>34</sup> or a stochastic-Liouville equation<sup>33</sup> treatment of the ESR. The nuclear-modulation problem (ignoring detailed spin relaxation) is adequately dealt with in the normal Hilbert space spanned by the spin basis functions as Mims has shown. In order to properly analyze the combined effects of nuclear modulation and spin relaxation required for the present 2D-ESR experiments (as well as conventional 1D ESEEM), we have recast the theory including relaxation in the full Liouville space. This has enabled us to properly assess how electron-spin relaxation, nuclear spin relaxation, and cross-relaxation affect the 2D spectra. We thereby indicate how such 2D-ESR experiments can provide both relaxation and structural information on radicals in a solid host.

In Sec. II, the experimental details are presented. The appropriate theory for these experiments is summarized in Sec. III, with a more detailed description of the derivation including spin relaxation given in the appendices. The results and discussion appear in Sec. IV and the conclusions are given in Sec. V.

## II. EXPERIMENT

All the experiments were performed on the 2D-FT-ESR spectrometer, which has been described elsewhere.<sup>16,20</sup> Additional modifications, as well as the bridged loop gap resonator (BLGR) utilized have been described by Patyal *et al.*<sup>26</sup> The use of a resonator with low  $Q \approx 45$  as well as short  $\pi/2$  pulses ( $\sim 5$  ns) permits wide-band coverage. In this work, we were able to achieve nearly uniform rotation of spins into the rotating  $xy$  plane over a  $\pm 100$  MHz bandwidth. This coverage is more than sufficient for the present study where the largest hf tensor component  $A_{yy} = 91$  MHz.

A significant increase in the rate of data acquisition was achieved over the previous studies by utilizing a computer-aided measurement and control (CAMAC) dual channel digitizer and averager (DSP Technology TRAQ-H model 2102SA) which samples at the rate of 200 megasamples/s (i.e., 5 ns dwell time). It was interfaced with a home-built synchronized clock, which allows for interleaving in successive sweeps to achieve an effective 1 ns resolution when needed. In the experiments reported here, we utilized a sweep rate of 1000 sweeps/s (for 256 data points). This yielded a total throughput rate of data acquisition that was about 15 times faster than we previously obtained using an HP54100A digitizing oscilloscope. Additional improvements, since completing these experiments, now permit sweep rates of 10 000 sweeps/s under otherwise identical conditions for an increase by about a factor of 150 over our original throughput rate.

Malonic acid crystals grown previously in our laboratory were used in this study. Some crystals were irradiated with 1 Mrad <sup>60</sup>Co  $\gamma$  radiation at the Ward Laboratory of Nuclear Engineering at Cornell University. A crystal of  $\sim 2$  mm  $\times$  1 mm  $\times$  0.5 mm, weighing 1.7 mg with well-defined faces and edges was used for data collection. Soon after irradiation, the cw-ESR spectrum shows several hf lines due to different kinds of radicals produced, but in aged samples (a couple of days after irradiation), only two strong lines persist. The strong doublet is due to the hf coupling of the  $\pi$  electron of the  $\alpha$  carbon with the  $\alpha$  proton of the C-H fragment of the  $\cdot\text{CH}(\text{COOH})_2$  radical of malonic acid.

The principal axis system of the hf tensor defined in Ref. 30 and the procedure to identify the axes described therein were used in this study. Figure 2 shows the principal axes ( $x, y, z$ ) of the hf tensor in relation to the structure of the malonic acid radical  $\cdot\text{CH}(\text{COOH})_2$ . The direction of the external static field  $B_0$  is defined by the angles  $\theta$  and  $\phi$ , where  $\theta$  is the angle between  $B_0$  and the  $z$  axis, and  $\phi$  is the angle between the  $x$  axis and the projection of  $B_0$  on the  $xy$  plane. The crystal was mounted on a specially designed delrin holder which was connected to a glass rod ( $\sim 2.5$  mm diameter). The other end of the glass rod was attached to a goniometer. The crystal was rotated successively about one of the three principal axes ( $x, y, z$ ) and measurements were made at several orientations.

At room temperature  $T_2^*$  is too short to allow the detection of a free induction decay (FID), but an echo decay can be easily measured. Figure 1 shows three types of experiments performed at each orientation. For the two-pulse SECSY-ESR experiment, the initial  $t_1$  was 110 ns due to the spectrometer dead time and 128 steps along  $t_1$  were used. The  $t_1$  step size was set either 5 or 10 ns depending on the spectral extent required for the nuclear modulation components which appear with respect to  $\omega_1$ . [That is, 5 (10) ns supplied a spectral extent of  $\pm 100$  ( $\pm 50$ ) MHz after FFT.] For each  $t_1$  step, the echo decay was collected along  $t_2$ , sampling 256 data points at a 1 ns interval. One important difference between our 2D technique and the conventional ESEEM is that we collect the echo shape starting at the peak of the echo as a function of  $t_2$  for each

$t_1$  value, which enables us to get the cw spectrum along  $\omega_2$  after Fourier transform.

In the three pulse stimulated-SECSY experiment, the evolution period  $t_1$  is kept constant and the mixing period  $T$  is stepped out; for each step in  $T$ , the stimulated echo was collected as a function of  $t_2$ . The initial  $T$  was 110 ns and 128  $T$  steps of either 5 or 250 ns step size were used. The larger step size supplied a  $T_{\max} = 32 \mu\text{s}$  which was needed for an accurate measurement of  $T_1 = 15\text{--}22 \mu\text{s}$ , the electron-spin lattice relaxation time, whereas the smaller step size (5 ns) was used to obtain sufficient spectral range ( $\pm 100$  MHz) required for the nuclear modulation components. In the echo-ELDOR experiment, the mixing period  $T$  is kept constant and the evolution period  $t_1$  is stepped out; for each step in  $t_1$ , the echo decay is collected as a function of  $t_2$ . The initial  $t_1$  was 110 or 130 ns depending on the signal to noise ratio. Here too 128  $t_1$  steps of 5 ns step size were used. At each orientation, three to five experiments were done for different values of the mixing time  $T$  to see any development of cross peaks as  $T$  increased.

Throughout all experiments, the phase cycling sequences suggested in Ref. 23 were used to remove the image peaks, transverse signals, and axial peaks. For example, the irradiated malonic acid crystal has five echo signals with comparable intensity in the three-pulse experiment. We could collect the stimulated echo signal selectively with phase cycling. Also, the  $S_z$  spectral combination<sup>23</sup> of the dual quadrature signal was used in the data processing, as is appropriate for an echo signal.

The data acquisition and processing methods are described in Refs. 26 and 27. The full 2D figures shown below were obtained directly by 2D-FFT of the original time-domain data and are the magnitude spectra. These data were also processed by linear prediction with singular value decomposition (LPSVD)<sup>20</sup> in order to get accurate relaxation times and frequencies as needed, and to enable us to produce contour plots of just the modulation peaks from the  $\alpha$  proton of malonic acid. The characteristic decay constants along  $t_1$  (SECSY) or  $T$  (stimulated SECSY) are summarized in Table I.

TABLE I. Characteristic relaxation constants for each orientation.

Orientation ( $\theta, \phi$ )	$T_2$ (ns) <sup>a</sup>	Decay constants along $T$ ( $\mu\text{s}$ ) <sup>b</sup>
(5°, 0°)	800	23.0
(30°, 0°)	812	21.1
(50°, 0°)	687	15.9
(5°, 90°)	1223	22.2
(45°, 90°)	1068	22.1

<sup>a</sup> $T_2$  is measured from SECSY experiment with 10 ns  $t_1$  step size.

<sup>b</sup>The decay constants along mixing time are measured from stimulated-SECSY experiments with 250 ns  $T$  step size.

order to deal properly with the 2D-ESR spectra from irradiated malonic acid, we extend the theory to include molecules with nonaxially symmetric  $g$  and hf tensors. We also include relaxation processes in a more explicit and detailed fashion.

In the general case, there is no axial symmetry in any of the tensors and the various tensors have a different principal axis system of their own. This general situation is summarized in the Schneider-Freed "User's Guide."<sup>35</sup> The following definitions will be used:

$$\bar{g} = \frac{1}{3} (g_{xx} + g_{yy} + g_{zz}) \frac{\beta_e B_0}{\hbar}, \quad \bar{a}_i = \frac{1}{3} (A_{xx}^i + A_{yy}^i + A_{zz}^i),$$

$$F = \frac{2}{3} \left[ g_{zz} - \frac{1}{2} (g_{xx} + g_{yy}) \right] \frac{\beta_e B_0}{\hbar},$$

$$D_i = \frac{2}{3} \left[ A_{zz}^i - \frac{1}{2} (A_{xx}^i + A_{yy}^i) \right],$$

$$F^{(2)} = \frac{1}{2} (g_{xx} - g_{yy}) \frac{\beta_e B_0}{\hbar}, \quad D_i^{(2)} = \frac{1}{2} (A_{xx}^i - A_{yy}^i),$$

where  $i$  is the index for the  $i$ th nucleus.

In high fields, the contribution of the nonsecular terms ( $S_{\pm}, S_{\pm} I_z, S_{\pm} I_{\pm}, S_{\pm} I_{\mp}$ ) is relatively small, so one need only use the secular and pseudosecular terms to determine the spectrum. Then the Hamiltonian in the case of one electron and several nuclei can be written as follows:

### III. THEORY

The Gamliel-Freed theory was developed to calculate the 2D-ESR spectra in the rigid limit.<sup>23</sup> In this work, in

$$\hat{H} = \left[ \bar{g} + F \frac{1}{2} (3 \cos^2 \beta - 1) + F^{(2)} \sin^2 \beta \cos(2\gamma) \right] S_z - \sum_i \omega_{ni} I_{zi} - \sum_i \frac{\gamma_e}{\hbar} \left\{ \left[ \bar{a}_i + D_i \frac{1}{2} (3 \cos^2 \beta^i - 1) \right. \right. \\ \left. \left. + D_i^{(2)} \sin^2 \beta^i \cos(2\gamma^i) \right] S_z I_{zi} + \left[ D_i \frac{3}{4} \sin \beta^i \cos \beta^i - D_i^{(2)} \frac{1}{2} \sin \beta^i \{ \cos \beta^i \cos(2\gamma^i) - i \sin(2\gamma^i) \} \right] S_z I_{+i} \right. \\ \left. + \left[ D_i \frac{3}{4} \sin \beta^i \cos \beta^i - D_i^{(2)} \frac{1}{2} \sin \beta^i \{ \cos \beta^i \cos(2\gamma^i) + i \sin(2\gamma^i) \} \right] S_z I_{-i} \right\},$$

where  $\Omega=(0,\beta,\gamma)$  represents the set of Euler angles describing the orientation of the  $g$  tensor relative to a laboratory-fixed frame and  $(\alpha^i,\beta^i,\gamma^i)$  are the Euler angles for the  $i$ th hf tensor. The spin operators are written in the lab frame. For the special case of a single nucleus interacting with the unpaired electron with the hf tensor having the same principal axes as the  $g$  tensor, the Hamiltonian is

$$\hat{H}=CS_z-\omega_n I_z+AS_z I_z+\frac{1}{2}BS_z I_+ +\frac{1}{2}B^*S_z I_- . \quad (3)$$

The coefficients in the Hamiltonian are now equal to

$$C=\bar{g}+F\frac{1}{2}(3\cos^2\beta-1)+F^{(2)}\sin^2\beta\cos(2\gamma),$$

$$A=-\frac{\gamma_e}{\hbar}[\bar{a}+D\frac{1}{2}(3\cos^2\beta-1)+D^{(2)}\sin^2\beta\cos(2\gamma)],$$

$$B=-\frac{\gamma_e}{\hbar}\{D_4^3\sin\beta\cos\beta-D^{(2)}\frac{1}{2}\sin\beta[\cos\beta\cos(2\gamma)-i\sin(2\gamma)]\}. \quad (4)$$

This is the form we specifically employ for irradiated malonic acid. Equation (3) is the typical form utilized by GF, but now the anisotropic tensor components appear in the coefficients  $A$ ,  $B$ , and  $C$  as given in Eq. (4). With this modest change, the GF theory again applies provided one is not interested in details of the spin relaxation.

GF included relaxation in an extremely simple fashion to avoid the necessity of dealing with the density matrix in the full Liouville space. Such a simplification permits one to treat the effects of the various nuclear spins in an additive fashion, as Mims has shown. A detailed analysis of relaxation, however, does require a full Liouville space analysis especially for dealing with cross-relaxation, exchange cross peaks in 2D-ESR, and also to distinguish the various  $T_2$ 's for the ESR and the NMR transitions. This complex matter is described in Appendix A for the case of an  $S=1/2$ ,  $I=1/2$  spin system, and an analytic analysis is provided after some simplifications are introduced. This permits us to write down analytic expressions for the two- and three-pulse 2D-ESR experiments described herein, which include detailed aspects of the spin relaxation.

### A. Two-pulse experiment—SECSY

The two-pulse SECSY experiment [cf. Fig. 1(a)] yields three types of signals—transverse, axial, and the echo or SECSY signal. The latter provides the nuclear modulation effects. Since each term shows distinctive dependence on the phases of the pulses as shown by GF, one may cancel the transverse and axial terms by phase cycling, leaving just the SECSY signal.

The SECSY signal is

$$S_{\text{SECSY}} \propto -ie^{i(2\phi_2-\phi_1)} \exp[-(2t_1+t_2)/T_{2e}] \times \left( k_+ \bar{A} + k_- F + (k/4) \sum_{i=1}^6 M_i \right), \quad (5)$$

$$\bar{A} = \cos(\omega_+ t_2), \quad F = \cos(\omega_- t_2), \quad M_1 = \cos(\omega_\alpha t_1 + \omega_- t_2),$$

$$M_2 = \cos(\omega_\beta t_1 - \omega_- t_2), \quad M_3 = \cos(\omega_\alpha t_1 + \omega_+ t_2),$$

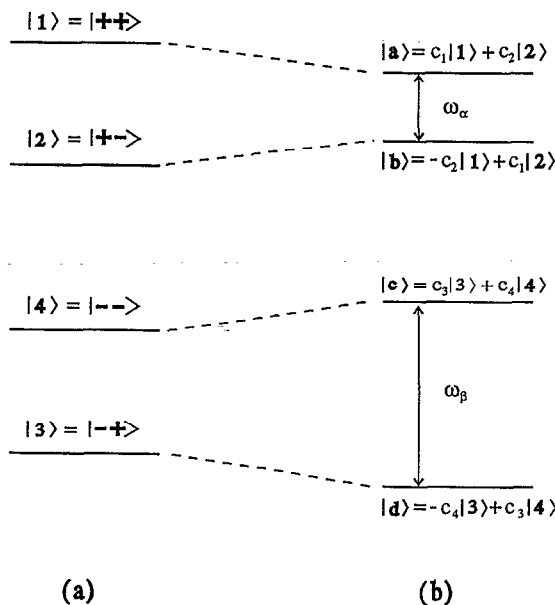


FIG. 3. The labeling of the spin eigenstates in the case of  $|A/2| > |\omega_n|$ . (a) The spin eigenstates in the  $|S_z I_z\rangle$  basis. The first and second signs represent  $M_S$  and  $M_I$ , respectively. These spin states are eigenstates in the case of  $B=0$ , i.e., no pseudosecular term [cf. Eq. (3)]. (b) The spin eigenstates in the presence of the pseudosecular term. Each state can be represented as a linear combination of the basis functions in the  $|S_z I_z\rangle$  basis. These eigenstates generate the  $\mathcal{H}_0$  basis.

$$M_4 = \cos(\omega_\beta t_1 + \omega_+ t_2),$$

$$M_5 = -\cos(2\omega_+ t_1 + \omega_+ t_2), \quad M_6 = -\cos(2\omega_- t_1 + \omega_- t_2),$$

and  $T_{2e}$  is the  $T_2$  for electron-spin transitions. Also  $\phi_1$  and  $\phi_2$  are the phases of pulses 1 and 2.

The parameters  $k$ ,  $k_+$ , and  $k_-$  appear in Table III in terms of the key frequencies  $\omega_\alpha$  and  $\omega_\beta$  (cf. Fig. 3)

$$\omega_\alpha = \left[ \left( \frac{A}{2} - \omega_n \right)^2 + \left| \frac{B}{2} \right|^2 \right]^{1/2},$$

$$\omega_\beta = \left[ \left( \frac{A}{2} + \omega_n \right)^2 + \left| \frac{B}{2} \right|^2 \right]^{1/2}$$

with  $\omega_\pm = \frac{1}{2}(\omega_\alpha \pm \omega_\beta)$ . Note that in Eq. (5), the symbol  $\bar{A}$  stands for the two allowed ESR resonances (i.e., allowed autopeaks obtained by rewriting the cosine as the sum of two exponentials),  $F$  stands for the two forbidden ESR resonances (i.e., forbidden autopeaks), and  $M_i$  ( $i=1,\dots,6$ ) for the 12 nuclear modulation (or coherence cross) peaks. The SECSY spectrum can be obtained by Fourier transforming Eq. (5) along both  $t_1$  and  $t_2$ . A schematic diagram for the SECSY experiment is shown in Fig. 4(a) for the case of  $|A/2| > |\omega_n|$  which is always the case regardless of the orientation angle in the  $\gamma$ -irradiated malonic acid single crystal. The case for  $|A/2| < |\omega_n|$  can be obtained from this by swapping  $\omega_+$  and  $\omega_-$  and by changing the sign of  $\omega_\beta$  (cf. Table III). Figure 4 illustrates the rich array of modulation peaks in a manner reminiscent of NMR COSY and SECSY involving spin-spin coupling. Note that Eq.

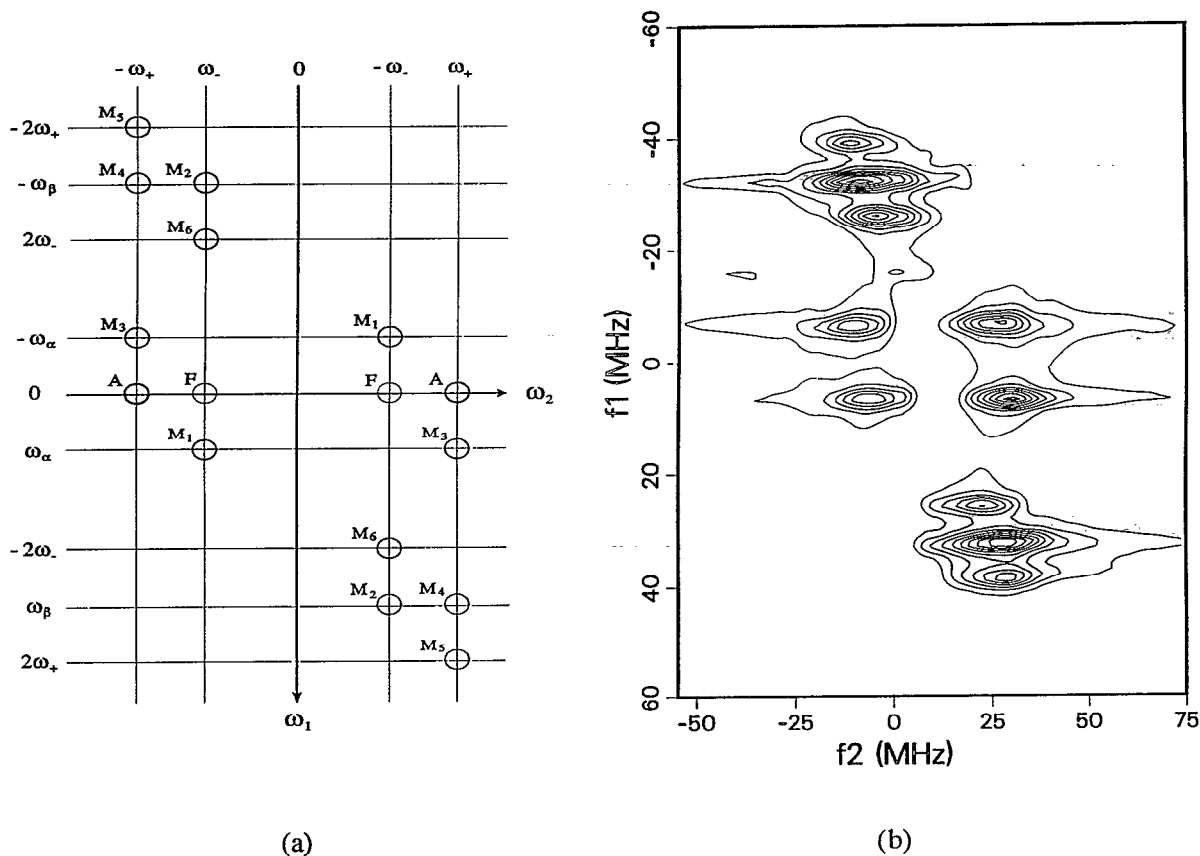


FIG. 4. (a) A schematic spectral diagram for the SECSY experiment; (b) experimental contour plot for the  $(30^\circ, 0^\circ)$  orientation [cf. Fig. 8(b)], where the intense allowed peaks along  $\omega_1 = 2\pi f_1 = 0$  have been removed.

(5) is exactly the equation previously obtained by GF, since relaxation appears in a very simple fashion in this two-pulse experiment, viz. all lines are broadened by the homogeneous  $T_{2e}$ .

In a real experiment, there is inhomogeneous broadening along the  $\omega_2$  axis, but the refocusing with respect to  $t_1$  in the echo experiment cancels out the inhomogeneous broadening along the  $\omega_1$  axis. Therefore one can obtain  $T_{2e}$  across the whole spectrum in a 2D-SECSY experiment. The Gaussian inhomogeneous broadening effect along  $\omega_2$  is included in our simulations as follows:

$$f_b(t_2) = f(t_2) \frac{1}{\sqrt{2\pi}\Delta} \int_{-\infty}^{\infty} \exp\left(-\frac{v^2}{2\Delta^2}\right) e^{-i2\pi vt_2} dv$$

$$= f(t_2) e^{-2(\pi\Delta t_2)^2}, \quad (6)$$

where  $f_b(t_2)$  is the broadened signal and  $\Delta$  is the Gaussian inhomogeneous broadening parameter in frequency units. Note that in the conversion of the internal angles  $(\theta, \phi)$  to the Euler angles, we get  $(\alpha, \beta, \gamma) = (0^\circ, -\theta, 0^\circ)$  for the  $zx$  quadrant ( $\phi = 0$ ; i.e., the rotation axis for the goniometer is parallel to the  $y$  axis);  $(\alpha, \beta, \gamma) = (0^\circ, \theta, 90^\circ)$  for  $zy$  quadrant and  $(\alpha, \beta, \gamma) = (0^\circ, 90^\circ, 180^\circ - \phi)$  for the  $xy$  quadrant. The intensity of the modulation peaks relative to the autopeaks is proportional to  $k/(4k_+)$ . This is plotted as a function of  $(\theta, \phi)$  and  $(A_{xx}, A_{yy}, A_{zz})$  in Figs. 5 and 6. It is found that

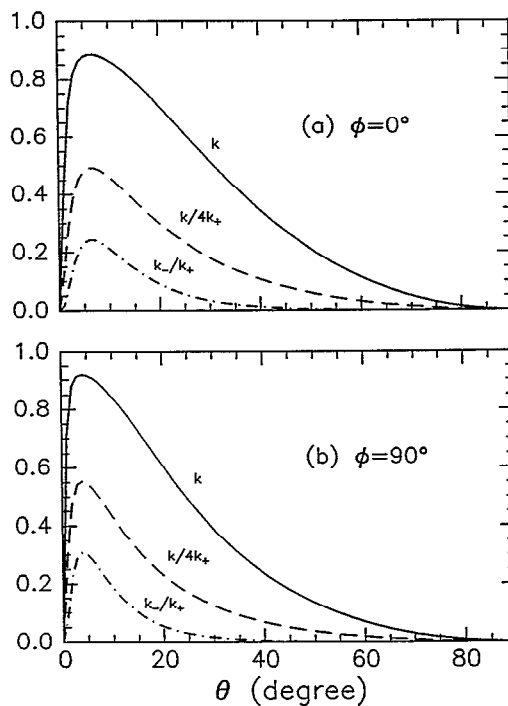


FIG. 5. The predicted variation of the intensity factors as a function of the orientation angle  $\theta$  in the plane of (a)  $\phi = 0^\circ$ ; (b)  $\phi = 90^\circ$ .  $A_{xx} = -61$  MHz,  $A_{yy} = -91$  MHz, and  $A_{zz} = -29$  MHz are used.

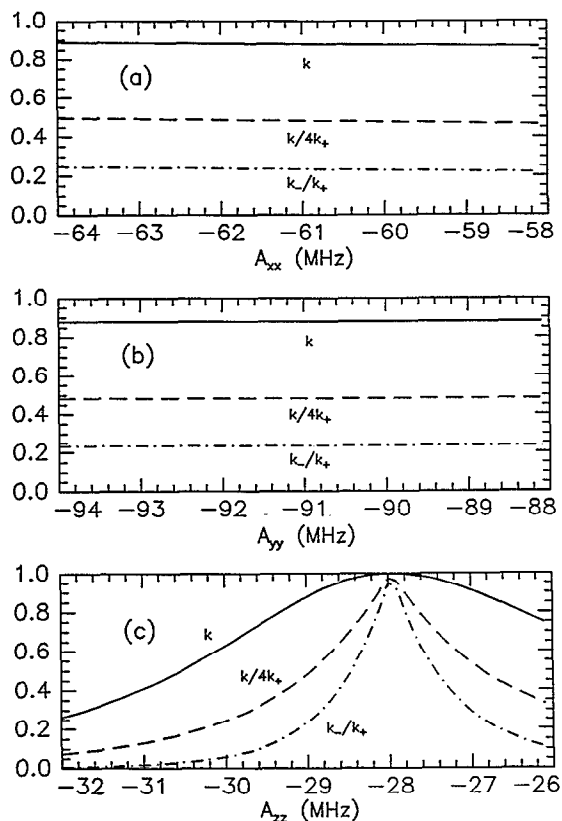


FIG. 6. The predicted variation of the intensity factors for  $\theta=5^\circ$ ,  $\phi=0^\circ$  as a function of the hf value (a)  $A_{xx}$ ; (b)  $A_{yy}$ ; (c)  $A_{zz}$ . For the fixed hf constants,  $A_{xx}=-61$  MHz,  $A_{yy}=-91$  MHz, and  $A_{zz}=-29$  MHz are used.

the intensity is quite sensitive to the  $A_{zz}$  value over the range  $0^\circ < \theta < \sim 15^\circ$ . If the crystal can be mounted accurately (possibly with the help of x-ray crystallography),  $(A_{xx}, A_{yy}, A_{zz})$  could be determined precisely from the intensity ratio. The modulation frequencies also show angular variation and are plotted in Fig. 7. They do not show much variation when the hf constants vary.

Simulated spectra based on the theory are shown in Fig. 8 for the  $zx$  quadrant and in Fig. 9 for the  $zy$  quadrant. As discussed in Sec. IV, where these predictions are com-

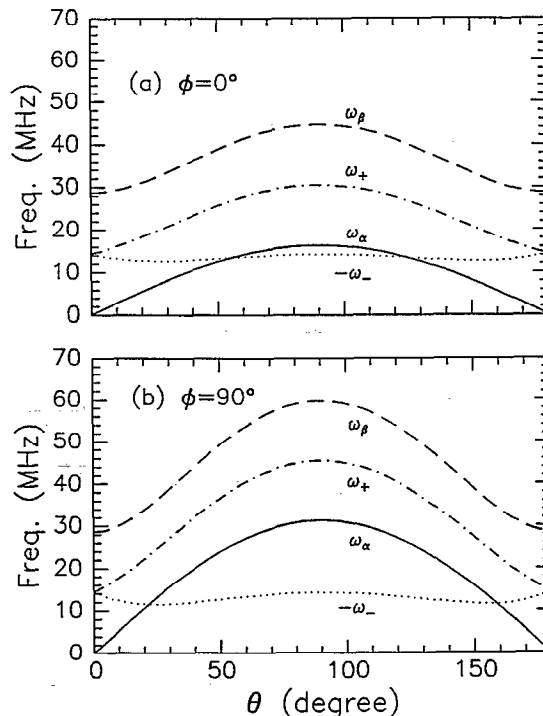


FIG. 7. The predicted angular variation of the modulation frequencies in the plane of (a)  $\phi=0^\circ$ ; (b)  $\phi=90^\circ$ .  $A_{xx}=-61$  MHz,  $A_{yy}=-91$  MHz, and  $A_{zz}=-29$  MHz are used.

pared with the experimental spectra, the agreement is quite good over all the orientation angles.

### B. Three-pulse experiments—echo-ELDOR and stimulated SECSY

In the three-pulse experiment (cf. Fig. 1), there are, in general, nine terms—four FID-like and five echoes.<sup>23</sup> The relevant signal in stimulated-SECSY or 2D-ELDOR is the stimulated echo signal. The phase cycling given in Table 4 of Ref. 23 removes all the transverse and axial peaks. The  $S_c$  combination of the dual quadrature signal then removes any residual FID ELDOR signal. Thus, only the stimulated echo signal is retained. The analysis based upon Appendix A yields

$$S_{\text{ELDOR}} \propto -i \exp[i(\phi_3 + \phi_2 - \phi_1)] \exp[-(2t_1 + t_2)/T_{2e}] \left( k_+ \bar{A} \{ \exp[-(2W_e + 2W_x)T] + \exp[-(2W_e + 2W_n + \omega_{\text{HE}})T] \} + k_+ E \bar{A} \{ \exp[-(2W_e + 2W_x)T] - \exp[-(2W_e + 2W_n + \omega_{\text{HE}})T] \} + k_- F \{ \exp[-(2W_e + 2W_x)T] + \exp[-(2W_n + 2W_x)T] \} + k_- E F \{ \exp[-(2W_e + 2W_x)T] - \exp[-(2W_n + 2W_x)T] \} + \frac{k}{4} \exp[-(2W_e + 2W_x)T] \sum_{i=1}^4 M_i + \frac{k}{4} e^{-T/T_{2n}} \sum_{j=5}^{12} N_j \right), \quad (7)$$

where

$$\bar{A} = \cos(\omega_+ t_2), \quad E \bar{A} = \cos(2\omega_+ t_1 + \omega_+ t_2), \quad F = \cos(\omega_- t_2), \quad E F = \cos(2\omega_- t_1 + \omega_- t_2), \quad M_1 = \cos(\omega_\alpha t_1 + \omega_- t_2), \\ M_2 = \cos(\omega_\beta t_1 - \omega_- t_2), \quad M_3 = \cos(\omega_\alpha t_1 + \omega_+ t_2), \quad M_4 = \cos(\omega_\beta t_1 + \omega_+ t_2), \quad N_5 = \cos(\omega_\alpha t_1 + \omega_\alpha T + \omega_- t_2),$$

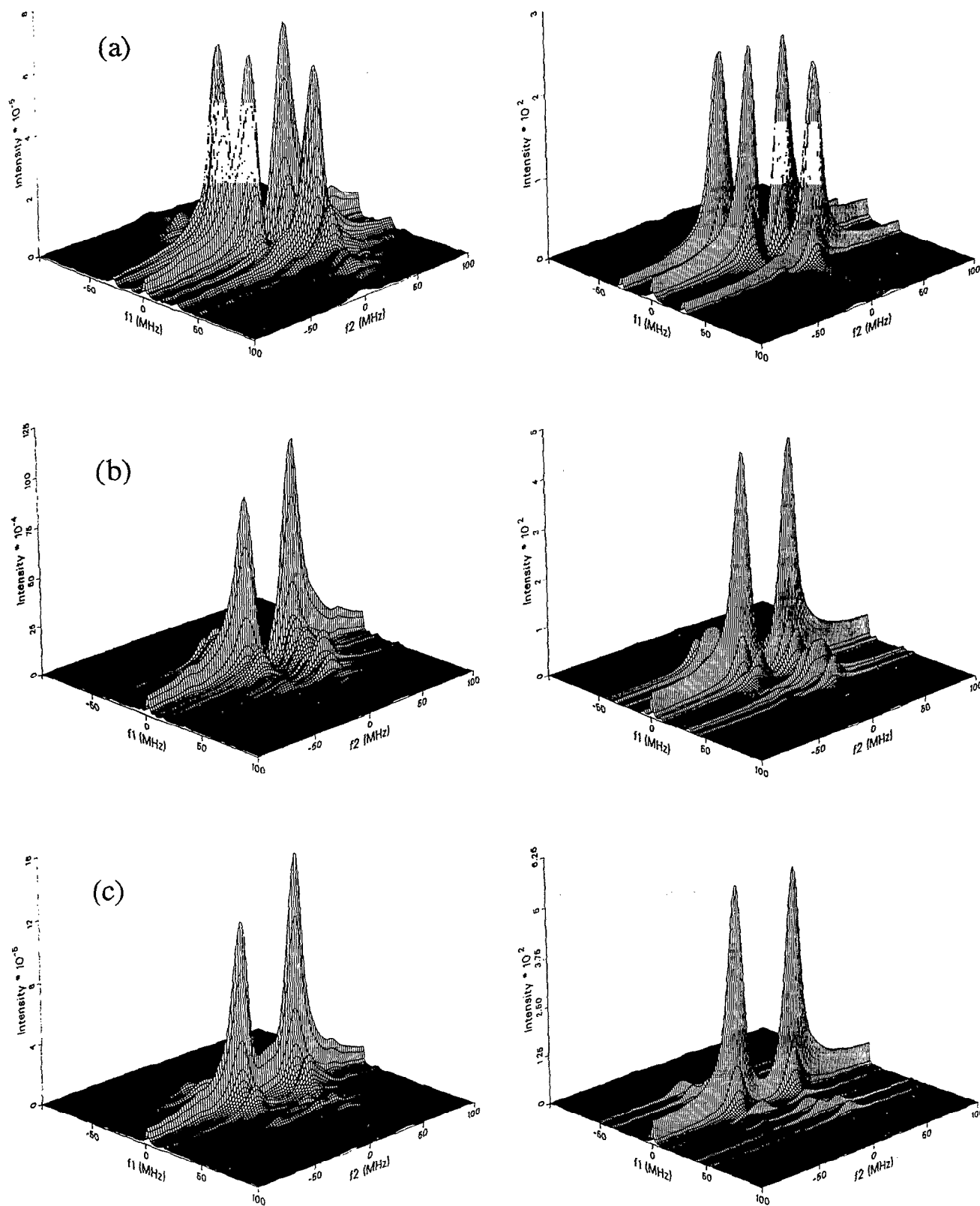


FIG. 8. Examples of experimental (left) and simulated (right) SECSY spectra at orientations of (a) ( $5^\circ, 0^\circ$ ); (b) ( $30^\circ, 0^\circ$ ); (c) ( $50^\circ, 0^\circ$ ). The measured  $T_2$  values in Table I and the inhomogeneous broadening width  $\Delta=4$  MHz are used in the simulation for each orientation.



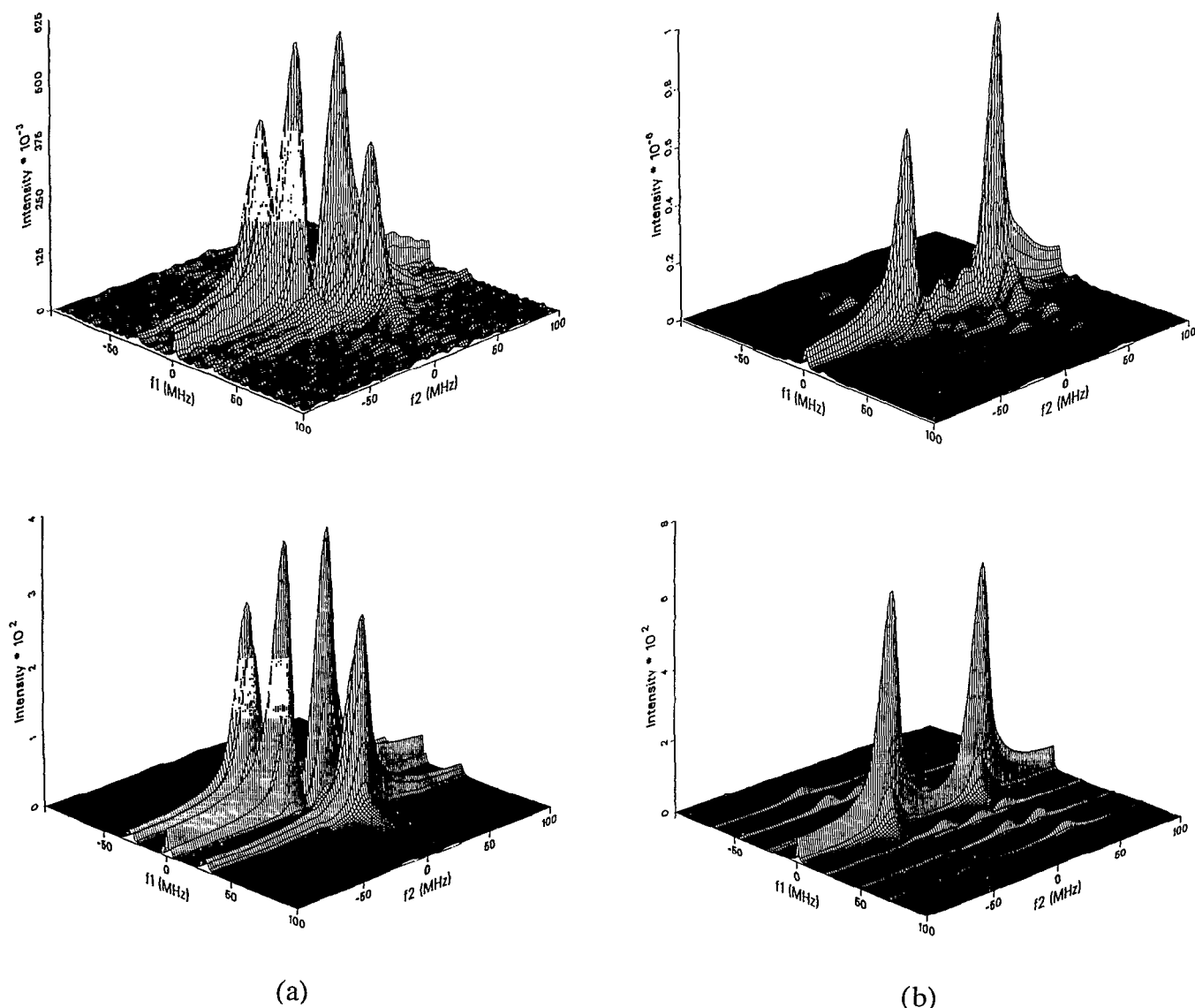


FIG. 9. Examples of experimental (upper) and simulated (lower) SECSY spectra at orientations of (a) ( $5^\circ, 90^\circ$ ); (b) ( $45^\circ, 90^\circ$ ). The measured  $T_2$  values in Table I and the inhomogeneous broadening width  $\Delta = 4$  MHz are used in the simulation for each orientation.

$$N_6 = \cos(\omega_\beta t_1 + \omega_\beta T - \omega_- t_2),$$

$$N_7 = \cos(\omega_\alpha t_1 + \omega_\alpha T + \omega_+ t_2),$$

$$N_8 = \cos(\omega_\beta t_1 + \omega_\beta T + \omega_+ t_2),$$

$$N_9 = -\cos(2\omega_- t_1 + \omega_\alpha T + \omega_- t_2),$$

$$N_{10} = -\cos(2\omega_- t_1 - \omega_\beta T + \omega_- t_2),$$

$$N_{11} = -\cos(2\omega_+ t_1 + \omega_\alpha T + \omega_+ t_2),$$

$$N_{12} = -\cos(2\omega_+ t_1 + \omega_\beta T + \omega_+ t_2),$$

and  $T_{2n}$  is the  $T_2$  for nuclear spin transitions. Also  $W_e$  and  $W_n$  are the lattice induced electron-spin flip and nuclear spin flip relaxation rates, with  $W_x$  a measure of the cross relaxation (cf. Ref. 11 and Appendix A);  $\omega_{HE}$  is the Heisenberg spin exchange<sup>11,36</sup> frequency (cf. Appendix A).

Equation (7) has the same allowed and forbidden autopeaks as the two-pulse SECSY expression of Eq. (5), but now there are exchange cross peaks labeled as  $E\bar{A}$  and  $EF$  which show up in the 2D-ELDOR experiment. Note that only the allowed autopeaks ( $\bar{A}$ ) and the allowed exchange peaks ( $E\bar{A}$ ) appear to be directly relaxed by the Heisenberg exchange process in the approximations made in Appendix A (which is not a general result). However,  $T_{2e}$  and  $T_{2n}$  both have contributions from  $\omega_{HE}$ .<sup>11,36</sup> There are two types of nuclear modulation cross peaks labeled  $M_i$  ( $i=1, \dots, 4$ ) and  $N_j$  ( $j=5, \dots, 12$ ). The  $M_i$  peaks come from spins along the  $z$  axis during the mixing period  $T$ , whereas for the  $N_j$  peaks, the spins are in the  $xy$  plane during this period, so they exhibit nuclear modulation with respect to  $T$ . In the limit of  $T=0$ , Eqs. (5) and (7) do become identical.

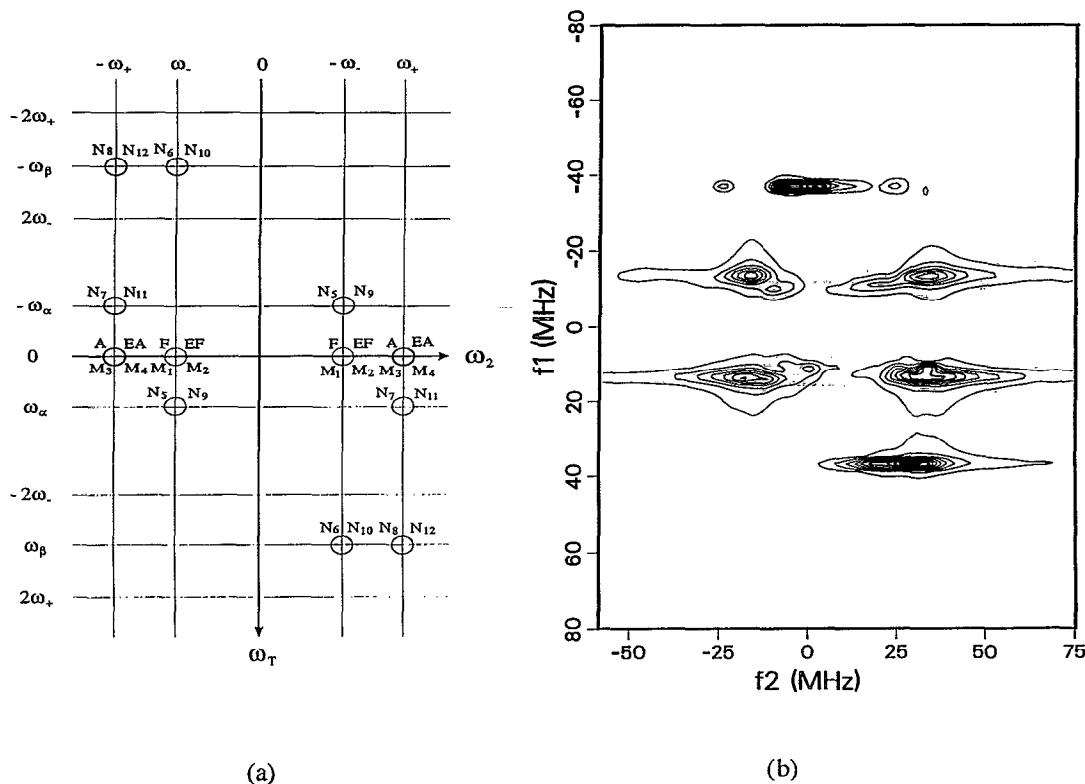


FIG. 10. (a) A schematic spectral diagram for stimulated SECSY; (b) experimental contour plot for the  $(50^\circ, 0^\circ)$  orientation [cf. Fig. 11(b)].

Note also the more complex relaxation than given by GF. The  $\bar{A}$  and  $F$  autopeaks each relax during the mixing period (i.e., vs  $T$ ) as the sum of two of the three normal relaxation modes found in Appendix A, whereas the exchange cross peaks  $E\bar{A}$  and  $EF$  relax according to differences in these modes. The  $M_i$  modulation peaks relax (vs  $T$ ) according to one of these modes, whereas the  $N_j$  peaks relax (vs  $T$ ) by  $T_{2n}$  since they are associated with nuclear transition terms during the mixing period. The GF theory<sup>23</sup> and its application by Patyal *et al.*<sup>27</sup> could not include such details of relaxation nor the exchange cross peaks. In order to get Eq. (7) to reduce to those of Refs. 23 and 27, one must let  $\omega_{HE} = W_n = W_x = 0$ ,  $T_1 = 1/2W_e$ , and  $T_2 = T_{2e} = T_{2n}$ , which represent oversimplifications of the relaxation (cf. Appendix B).

### 1. Stimulated SECSY

Fourier transformation of Eq. (7) along  $T$  and  $t_2$  gives the stimulated-SECSY spectrum, whose schematic diagram is shown Fig. 10(a) for  $|A/2| > |\omega_n|$ . The intensity of all the peaks at  $\omega_T = \pm\omega_\alpha$  becomes zero if  $\omega_\beta t_1 = 2n\pi$  with  $n$  an integer. This is the so-called suppression effect in conventional ESEEM. This suppression effect is seen to be due to pairs of  $N$ -type modulation peaks with opposite signs appearing at the same location in the 2D spectrum (i.e.,  $N_5$  and  $N_9$ , and  $N_7$  and  $N_{11}$ ).  $t_1$  is preset to avoid the suppression effect in the experiment. A series of simulated spectra are shown in Fig. 11. They are compared to exper-

imental spectra (also in Fig. 11) in Sec. IV and are found to agree well over all the orientation angles.

### 2. Echo-ELDOR

The echo-ELDOR spectrum is obtained by Fourier transform of Eq. (7) along  $t_1$  and  $t_2$  and its illustrative diagram is shown in Fig. 12. The general pattern is the same as that for SECSY (cf. Fig. 4), but now there are pairs of modulation terms (but all of the same sign) contributing to the nuclear modulation cross peaks, i.e., there is no suppression effect. However, the relative intensities in 2D-ELDOR contain considerable information on the relaxation parameters. Another important difference is the existence of exchange cross peaks ( $E\bar{A}$  and  $EF$ ) which grow in with  $T$  due to  $W_n$  or  $W_x$ . If  $W_n > W_x$  the exchange cross peaks between the allowed transitions ( $E\bar{A}$ ) have the same sign as the autopeak. If  $W_n < W_x$  they have the opposite sign. Therefore the source of the exchange cross peaks determines the phase, which can be obtained before taking the magnitude spectrum. However, the observation of  $E\bar{A}$  is somewhat obscured because the nuclear modulation peaks  $N_{11}$  and  $N_{12}$  appear at the same positions and their phases do depend on  $T$ .

Simulated spectra, selected to compare with experimental ones in Fig. 13, are shown in Fig. 14. They are a good example of how the relaxation parameters can be derived from the echo-ELDOR spectra with different mixing time. It is convenient to use a relative intensity scale since the intensity of the allowed autopeak ( $\bar{A}$ ) itself is a

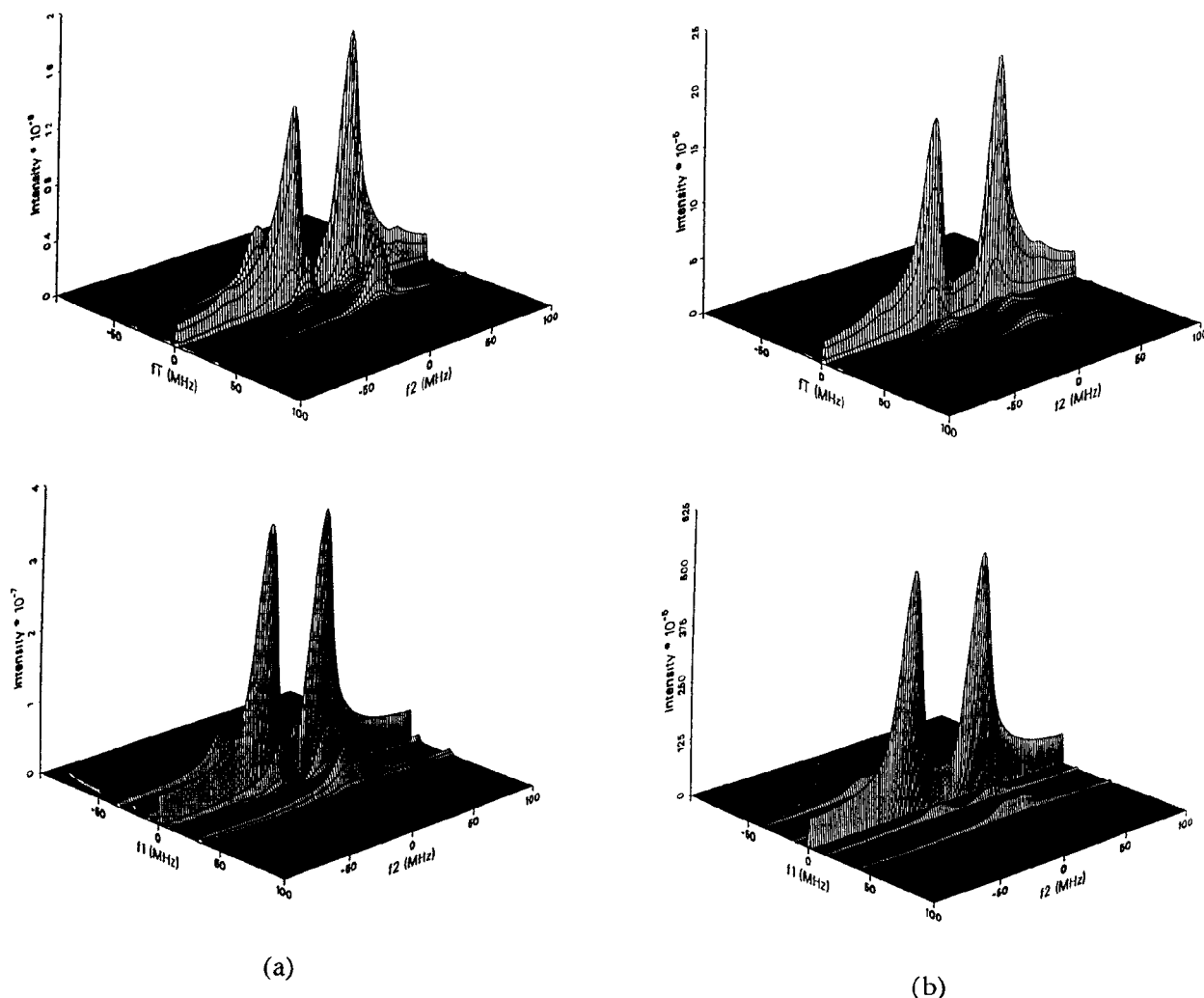


FIG. 11. Examples of experimental (upper) and simulated (lower) stimulated SECSY spectra at orientations of (a)  $(30^\circ, 0^\circ)$ ; (b)  $(50^\circ, 0^\circ)$ . The parameters used in the simulation are  $T_{2e}=900$  ns,  $T_{2n}=22$   $\mu$ s,  $W_e=1.67 \times 10^4$  s $^{-1}$ ,  $W_n=7.14 \times 10^3$  s $^{-1}$ ,  $W_x=6.17 \times 10^3$  s $^{-1}$ , and  $\Delta=5$  MHz.

function of  $T$  (i.e., let the intensity of peak  $\bar{A}$  be unity). The development of the exchange cross peaks ( $E\bar{A}$ ) can be clearly seen as the mixing time  $T$  increases.

## IV. RESULTS AND DISCUSSION

### A. SECSY

We show in Figs. 8 and 9 a series of SECSY spectra obtained for a range of values of orientation angles  $\theta$  and  $\phi$ . These 2D spectra were obtained by a 2D-FFT of the raw experimental data. One sees that the modulation peaks vary dramatically as a function of orientation as predicted (cf. Figs. 5 and 7). The modulation peaks are a maximum for a small value of  $\theta \cong 5^\circ$  (either for  $\phi=0^\circ$  or  $90^\circ$ ) and are comparable in magnitude to the autopeaks. The complex modulation pattern predicted (cf. Fig. 4) becomes more differentiated for larger values of  $\theta \sim (30^\circ-50^\circ, \phi=0^\circ$  or  $90^\circ)$ , where the intensities of the modulation peaks are significantly less than those of the autopeaks and the separation of the peaks  $\omega_\alpha$  is large. Also included for compar-

ison are the simulated spectra appropriate for the particular orientation, the measured  $T_{2e}$  (which is found to be orientation dependent, as expected), and the observed inhomogeneous broadening. The simulations do not include contributions from secondary radical species known to be present,<sup>30</sup> nor the nuclear modulation from matrix protons which appear along  $f_1$  at the nuclear Zeeman frequency  $\omega_n \cong 14$  MHz. The general agreement between experiment and theory is seen to be excellent with respect to the features due to the  $\alpha$  proton of malonic acid, and they are the dominant features. In order to more clearly discern the characteristic patterns of the modulation peaks diagrammed in Fig. 4(a), we display a contour plot for one of the orientations ( $\theta=30^\circ, \phi=0^\circ$ ) in Fig. 4(b). This contour was processed by applying LPSVD to the data of Fig. 8(b). This enabled us to remove the dominant autopeaks which would otherwise overwhelm the contour plots, as well as the broad contributions presumably from the matrix protons (at  $f_1 = \pm 13.5$  MHz). Indeed, one discerns all the modulation peaks with the expected overlaps of

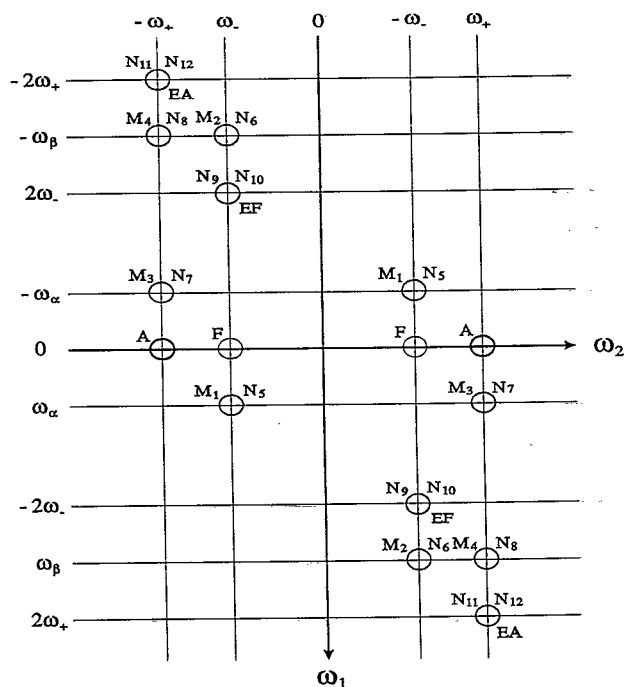


FIG. 12. A schematic spectral diagram for the echo-ELDOR experiment.

peaks that are nearby. The two pairs of  $M_1$  and  $M_3$  peaks are clearly distinguishable. The two groupings of the  $M_2$ ,  $M_4$ ,  $M_5$ , and  $M_6$  peaks are also clearly discernible. Although overlapped, one sees the  $M_5$  and  $M_6$  peaks, while the  $M_2$  and  $M_4$  peaks, both of which occur at  $\omega_1 = \omega_\beta$ , form a broad doubled-width line as expected since the inhomogeneous broadening affects only the  $\omega_2$  direction (and not the  $\omega_1$  direction in this echo-based experiment).

### B. Stimulated SECSY

In Fig. 11, we show stimulated SECSY spectra for several orientations as well as the respective simulations. Again, the characteristic features predicted for the  $\alpha$  proton of malonic acid are clearly discerned in the experimental spectra. We further examine the modulation patterns by first using LPSVD to remove the autopeaks along  $\omega_1 = 0$ . As can be seen from Fig. 10(a), the  $M_i$  ( $i=1, \dots, 4$ ) modulation peaks are degenerate with the autopeaks and are therefore not observed. The remaining spectrum showing the  $N_j$  ( $j=5, \dots, 12$ ) modulation peaks is displayed in the contour plot of Fig. 10(b) [based on the data of Fig. 11(b)] corresponding to  $\theta=50^\circ$ ,  $\phi=0^\circ$ . One clearly discerns the two pairs of  $N_5+N_9$  and  $N_7+N_{11}$  peaks at  $\omega_T = \pm\omega_\alpha$  as well as the broadened-out groupings of  $N_6+N_{10}$  with  $N_8+N_{12}$ . Because of the suppression effect in stimulated SECSY, the intensities for each grouping are influenced differently for the particular value of  $t_1$  one utilizes (see below), and this influences somewhat the pattern seen in the contour plot.

The LPSVD data analysis of the spectra with 250 ns  $T$  step size show that we have only a single exponential decay along the  $T$  axis for the allowed autopeaks ( $\bar{A}$ ), whereas the theory predicts the superposition of two distinctive decays  $1/(2W_e+2W_x)$  and  $1/(2W_e+2W_n)$  within our relaxation scheme; (we assume  $\omega_{HE} \cong 0$ , i.e., our crystal is dilute enough to neglect concentration-dependent spin relaxation processes). This might be due to the fact that those two decays are very close and LPSVD cannot distinguish their difference. However, Eq. (7) shows stimulated-SECSY gives direct information on the normal modes of relaxation which cannot in general be ascribed to a simple  $T_{1e}$ .

### C. 2D-ELDOR

We show in Fig. 13 a sequence of 2D-ELDOR experiments for a particular orientation ( $\theta=30^\circ$ ,  $\phi=0^\circ$ ). First of all, we wish to comment that the 2D-ELDOR spectra due to the nuclear modulation are very similar to the SECSY spectrum for the same angle, although differences in intensity do arise from the different dependences on  $T$  of the various modulation peaks and their different relaxation dependences [cf. Eq. (7)]. In general, we do observe that the 2D-ELDOR spectra are similar to the SECSY spectrum for the equivalent crystal orientation. Thus, we only display in Fig. 13 spectra for a single orientation, but as a function of the mixing time,  $T$ . We display in Fig. 14 a set of simulations of the 2D-ELDOR experiment for the range of mixing times, and again generally good agreement between theory and experiment is observed. The changes with  $T$  are seen to be rather subtle. We can perhaps observe them more clearly in the contour plot format wherein the dominant autopeaks along  $\omega_1=0$  are removed by LPSVD. A series of contours are shown in Fig. 15. These do indeed display the basic SECSY pattern seen in Fig. 4. As one looks with greater detail, one observes some significant variations with  $T$ . The intensity of the forbidden peaks is much smaller than that of the allowed peaks ( $k_-/k_+ \cong 0.03$ ) for this orientation (cf. Fig. 5). Therefore we can ignore the contribution of  $F$  and  $EF$ . The modulation frequency  $\omega_\beta$  from the experimental spectra is  $\sim 32$  MHz, which implies the actual orientation is approximately ( $25^\circ, 0^\circ$ ). The frequency  $\omega_\alpha$  is a little obscured by the broadening effect of the autopeaks. Once each term in Eq. (7) is classified according to the position of the peak by the following equation and its equivalents:

$$\begin{aligned} & \cos(2\omega_-t_1 + \omega_\alpha T + \omega_-t_2) \\ &= \frac{1}{2}e^{i\omega_\alpha T} \exp[i(2\omega_-t_1 + \omega_-t_2)] + \frac{1}{2}e^{-i\omega_\alpha T} \\ & \quad \times \exp[-i(2\omega_-t_1 + \omega_-t_2)] \end{aligned} \quad (8)$$

the calculation of the absolute intensities at each peak position including the relaxation constants shows

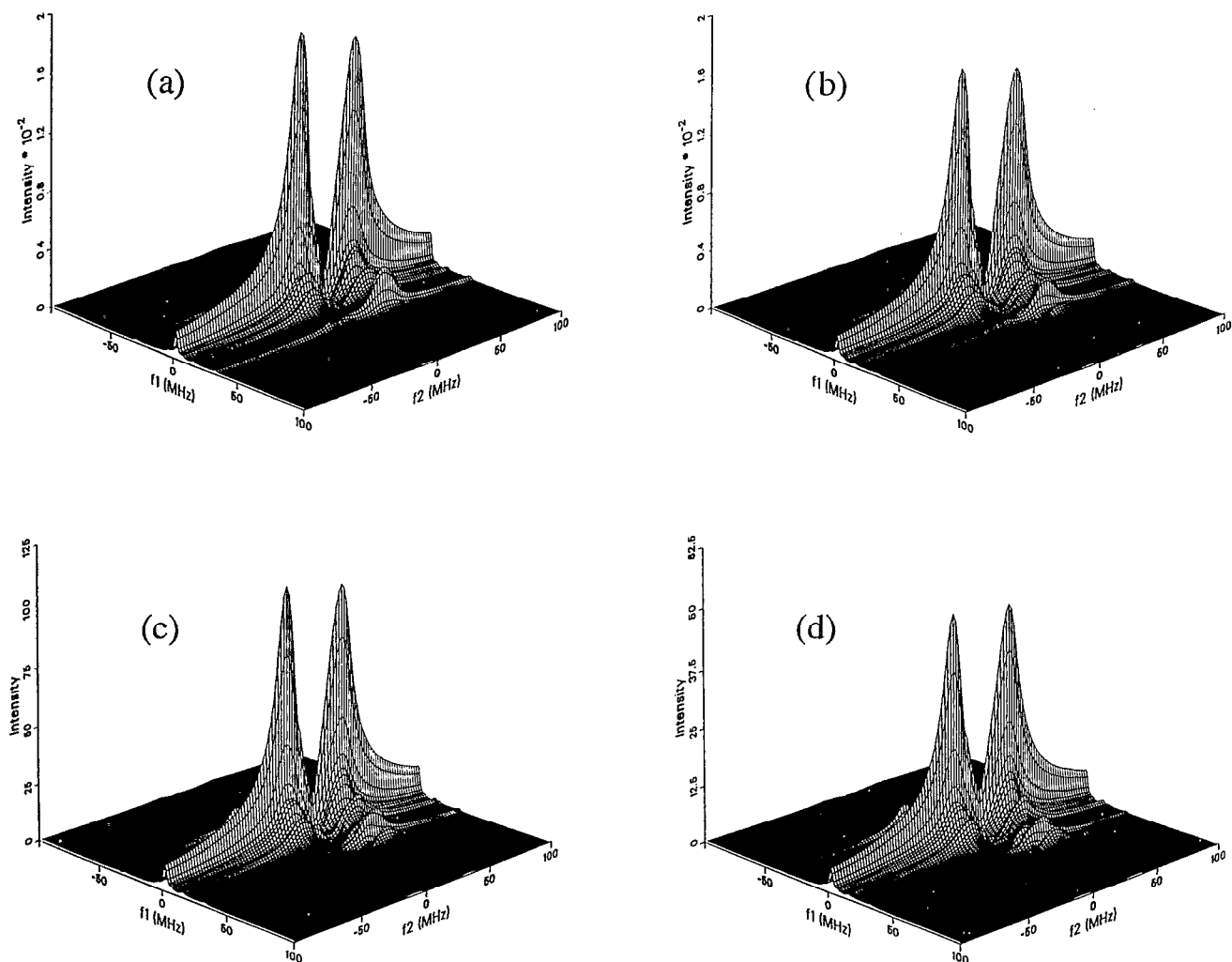


FIG. 13. Stacked plots of the experimental echo-ELDOR spectra at the orientation of  $(30^\circ, 0^\circ)$  with mixing time (a)  $T=5$ ; (b) 20; (c) 40; (d)  $60 \mu\text{s}$ .

$$M_1 + N_5 \text{ or } M_3 + N_7 \text{ depends on } \frac{k}{4} \{ \exp[-(2W_e + 2W_x)T] + e^{-T/T_{2n}} e^{i\omega_\alpha T} \},$$

$$M_2 + N_6 \text{ or } M_4 + N_8 \text{ depends on } \frac{k}{4} \{ \exp[-(2W_e + 2W_x)T] + e^{-T/T_{2n}} e^{i\omega_\beta T} \}, \quad (9)$$

$$N_9 + N_{10} \text{ depends on } -\frac{k}{8} e^{-T/T_{2n}} (e^{i\omega_\alpha T} + e^{-i\omega_\beta T}),$$

$$N_{11} + N_{12} + E\bar{A} \text{ depends on } -\frac{k}{8} e^{-T/T_{2n}} (e^{i\omega_\alpha T} + e^{i\omega_\beta T}) + k_+ \{ \exp[-(2W_e + 2W_x)T] - \exp[-(2W_e + 2W_n)T] \}.$$

The intensity of the  $N_9 + N_{10}$  pair can be different from that of the  $N_{11} + N_{12}$  pair due to their different phase dependence with respect to  $T$  in the ELDOR experiment. Note that for  $T=5 \mu\text{s}$ , the  $N_{11} + N_{12}$  pair, which is degenerate with the allowed exchange cross peak ( $E\bar{A}$ ) appears to be missing. Such an effect is due likely to the opposite

phase of  $N_{11} + N_{12}$  with  $E\bar{A}$ . As  $T$  increases, one observes that this (composite) peak grows in relative to the others, so that by  $T=60 \mu\text{s}$ , it has become a dominant feature. This can only be explained by the growing in of the exchange peak relative to the modulation peaks as predicted by Eq. (7) provided there is a substantial  $W_n$  or  $W_x$ .

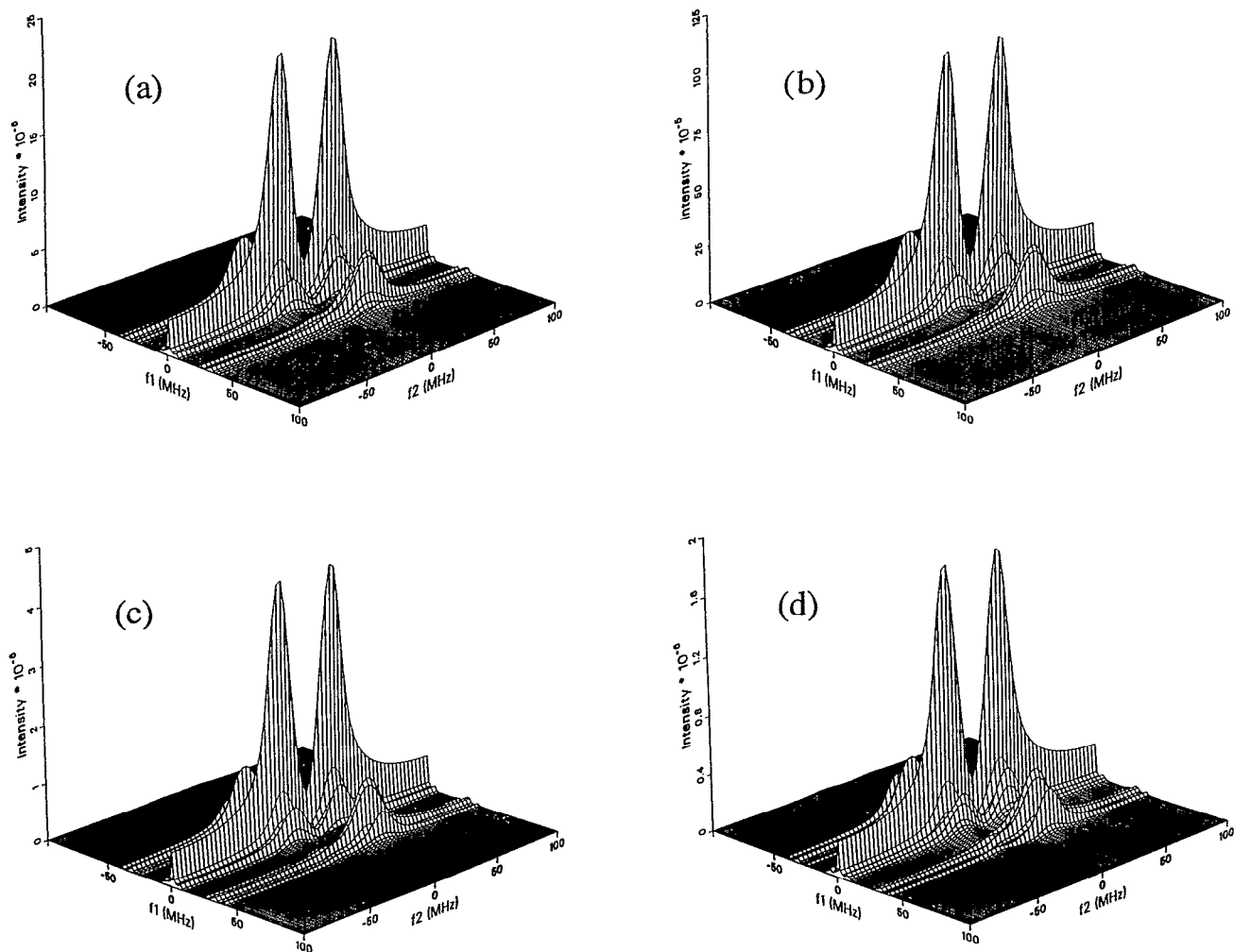


FIG. 14. The simulated echo-ELDOR spectra corresponding to Fig. 13. The same parameters as in Fig. 11 are used.

Note that the phases of the complex number quantities  $e^{i\omega T}$  are rapidly oscillating. For  $\omega_\beta = 32$  MHz,  $e^{i\omega_\beta T}$  oscillates 32 cycles over a  $1 \mu\text{s}$  period. This shows the intensity of the  $N_j$  peaks would be oscillating. When the intensity of the  $M_i$  terms and the exchange term is comparable to that of  $N_j$  terms, the intensity of the peaks where  $M_i + N_j$  and  $N_{11} + N_{12} + E\bar{A}$  terms appear would also be rapidly oscillating. It is thus very difficult to obtain the phase of each peak with only five different mixing times since the intensity is a function of the relaxation constants and the modulation frequencies. Therefore, a slight uncertainty in the modulation frequencies from the value of the magnetic constants and the orientation of the single crystal leads to substantial uncertainty in the determination of the relaxation constants. Thus it is first important to measure the modulation frequencies accurately (and to accurately orient the single crystal) prior to analyzing the spin relaxation. Then a series of experiments with many mixing times stepped by  $\sim 200$  ns, so that the contribution of the

exchange peak is almost constant over several steps, would be useful to determine the phase of the peaks more accurately, since the modulation frequencies  $\omega_\alpha$  and  $\omega_\beta$  are about 7 and 32 MHz, respectively.

Even though the present work was not performed in this fashion, we were able to utilize our LPSVD data analysis as well as measurements of the peak heights to constrain our choices of relaxation parameters, according to the following observations:

(i) The existence of exchange cross peaks is clearly indicated by the following: The peaks at  $(\omega_1, \omega_2) = (\pm 2\omega_+, \pm \omega_+)$  given by an  $(N_{11} + N_{12} + E\bar{A})$  increase with  $T$ , for  $T > 20 \mu\text{s}$ , relative to the  $\bar{A}$  peaks. [Note that the  $(N_9 + N_{10})$  peaks do not show much variation relative to the  $\bar{A}$  peaks.]

(ii) At no time for  $T \leq 60 \mu\text{s}$  do the  $E\bar{A}$  peaks become dominant over the  $(N_{11} + N_{12})$  peaks. Given the variation in phase of these latter peaks with  $T$  [cf. Eq. (9)] and other instrumental contributions to the phase of the various

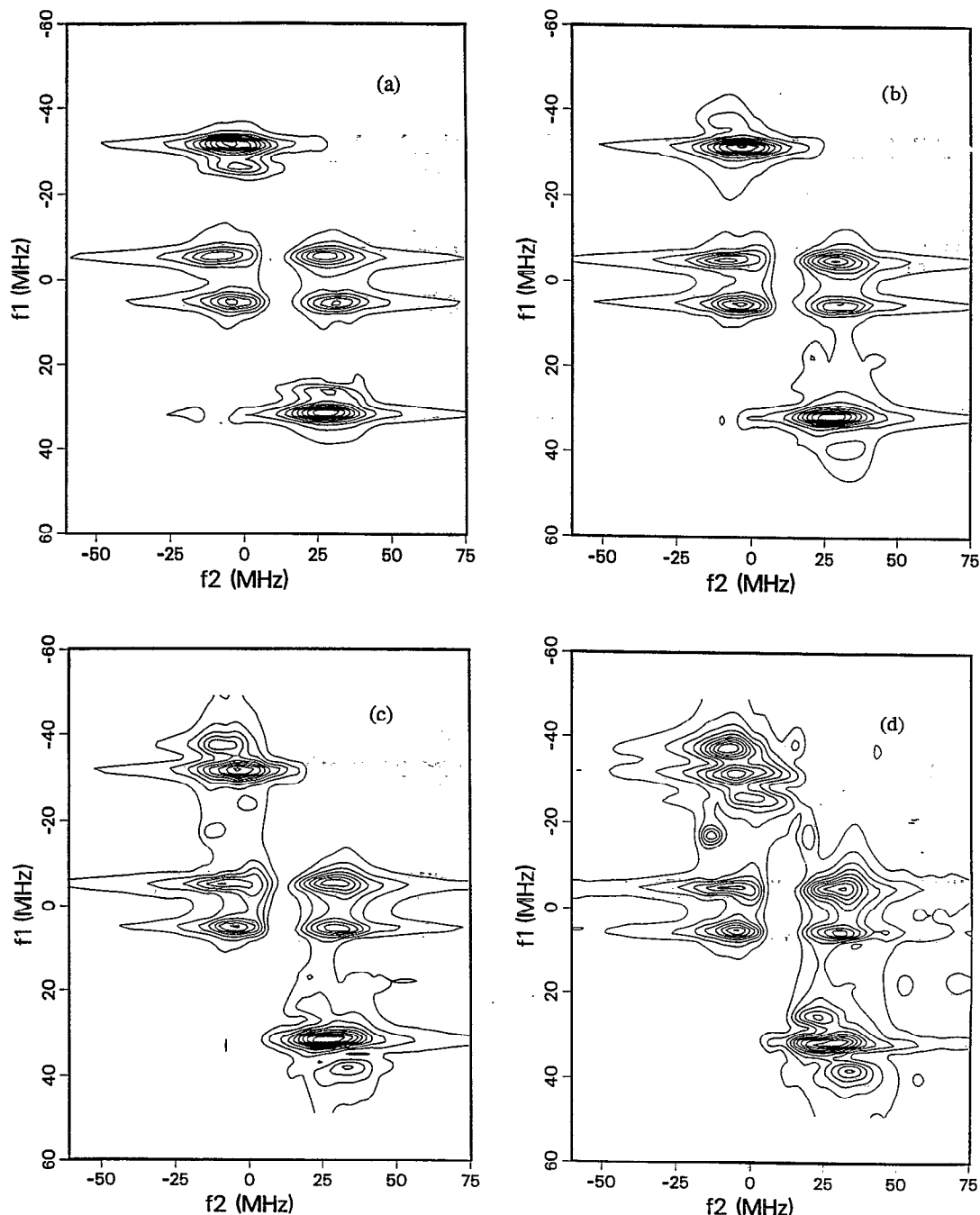


FIG. 15. Contour plots of the echo-ELDOR spectra shown in Fig. 13.

peaks,<sup>26</sup> we could not reliably determine the sign of  $E\bar{A}$  relative to  $\bar{A}$ .

(iii) However, we do observe that the relative intensity of the exchange peak (i.e.,  $|E\bar{A}/\bar{A}|$ ) grows to  $\sim 0.06$ – $0.10$  in  $T=60 \mu\text{s}$ . Since  $|E\bar{A}/\bar{A}| = [1 - \exp(-|2W_n - 2W_x|T)] / [1 - \exp(-|2W_n - 2W_x|T)]$ , then we roughly estimate  $|2W_n - 2W_x| \sim 2$ – $3 \times 10^3 \text{ s}^{-1}$ .

(iv) The stimulated SECSY experiment yielded a decay constant of  $21.1 \mu\text{s}$  which should be close to the similar

values of  $1/(2W_e + 2W_x)$  and  $1/(2W_e + 2W_n)$ .

The  $T_2$ 's were constrained by

(v) the SECSY experiment yielding  $T_{2e} = 900 \text{ ns}$ ;

(vi) the ratio of the peaks ( $N_9 + N_{10}$ ) vs ( $M_2 + N_6$ ) grows with increasing  $T$ , corresponding to the  $M$  vs  $N$  peak ratio decreasing with  $T$ . This yields  $T_{2n}^{-1} < (2W_e + 2W_x)$ , but the relation  $T_{2n}^{-1} = T_{2n}^{\prime-1} + W_e + W_n + W_x$  [cf. Eq. (B2)] yields  $T_{2n}^{-1} > W_e + W_n + W_x$ . These two inequalities provide bounds for  $T_{2n}^{-1}$ .

TABLE II.  $T_2$  variation of the main peaks for different mixing time.

Orientation	Average $T_2$ of the two main peaks (ns)						
	Mixing time ( $\mu$ s)	0 <sup>a</sup>	5	20	30	40	60
(30°,0°)		908	763	588	549	503	445
(50°,0°)		928	718	452	439	402	317

<sup>a</sup> $T_2$  for zero mixing time is obtained from the SECSY experiment.

<sup>b</sup>All the experiments are performed with 5 ns  $t_1$  step size.

The parameters were varied within these constraints to fit the set of 2D-ELDOR spectra for different  $T$ . This enabled us to obtain  $W_e$  more precisely and from that estimates of the absolute values of  $W_x$  and  $W_n$ , as well as  $T_{2n}^{-1}$ . We arbitrarily chose  $W_n > W_x$  (but we could fit the 2D spectra equally well with  $W_x > W_n$ ). These results are given in the caption to Fig. 11.

#### D. 2D-ELDOR “motional” cross peaks

Patyal *et al.*<sup>27</sup> showed in their study of a nitroxide in a glycerol host that the 2D-ELDOR main peaks broaden with mixing time  $T$ . This is consistent with the development of “rotational cross peaks” as the molecule slowly reorients during the mixing time. That is, one is observing how the slow motions evolve in real time.

We have examined the time evolution of the apparent widths of the main peaks as a function of  $T$  for several crystal orientations. Indeed we do find a broadening as  $T$  increases which we can ascribe to some slow dynamical process in the single crystal. {Note that we use the fact that 2D-ELDOR at  $T=0$  is exactly the same as the SECSY-ESR experiment [cf. Eqs. (5) and (7)] to obtain the former.} We provide these results in Table II. Note that the modulation peaks are, in general, observed to broaden with  $T$  (cf. Fig. 15), but because of their lower intensity and overlap effects, our analysis of them proved less accurate. We find over the 60  $\mu$ s period that the intrinsic  $T_2$  of 908 ns broadens in the 2D-ELDOR experiment to 445 ns for the orientation of ( $\theta=30^\circ$ ,  $\phi=0^\circ$ ). This corresponds to an increase in “width” of about 0.34 MHz. The same analysis for ( $\theta=50^\circ$ ,  $\phi=0^\circ$ ) leads to a 0.26 MHz increase. This is to be compared with the inhomogeneously broadened hf width along  $\omega_2$  of about 4–5 MHz.

In speculating about the nature of the processes being observed, we wish to note that there are two classes of dynamic processes which could have the correct qualitative behavior. First are the solid-state “spin-dynamics” processes related to spectral diffusion, in particular, spin diffusion of the proton spins<sup>37–39</sup> in the malonic acid crystal. The weak dipolar coupling of the unpaired electron spin of the malonic acid radical with surrounding matrix protons will lead to a distribution of local frequency shifts that would contribute to the inhomogeneous broadening of the cw ESR line. However, spin diffusion within the manifold of matrix protons will modulate these local frequency shifts in a time-dependent fashion. If this modulation were not

too fast, then this phenomenon could lead to a broadening of the 2D-ELDOR peaks as  $T$  increases, i.e., a spectral diffusion. However, Mims<sup>37</sup> has argued that one should be in the “narrowing” regime, since estimates of the frequency shifts caused by the nuclear spin diffusion are small compared to the rate of nuclear spin flip flops, which are the basis of the spin-diffusion process. Such “motional averaging” would contribute to the homogeneous  $T_{2e}$ <sup>37,38</sup> but would not contribute to the broadening of the 2D-ELDOR peaks vs  $T$ . But the detailed mechanism is a complex one, and more careful analyses suggest otherwise,<sup>37,39</sup> since the nuclear-spin flip-flop times (estimated as  $\sim 10^{-3}$ – $10^{-4}$  s) are long compared to the time scale of the experiment. As formulated for experiments at low temperatures (77 K or lower<sup>38,39</sup>), this spectral diffusion process is temperature independent, but at room temperature, one might anticipate a thermal enhancement of nuclear spin flip-flop rates.

At higher electron spin concentrations, there are equivalent spectral diffusion processes whereby nearby electron spins cause local frequency shifts of each electron spin, and these frequency shifts are modulated either by electron-spin diffusion within the electron-spin manifold (“ $T_2$ -type samples”), or by  $T_{1e}$  processes (“ $T_1$ -type samples”).<sup>38,39</sup> In  $T_2$ -type samples, mutual flip flops of electron spins can also be effective,<sup>38,39</sup> as outlined in Appendix A. These and related electron-spin-spin processes have been studied in malonic acid by electron-spin echoes at low (77 K) temperatures.<sup>40</sup> There are, however, subtle distinctions which affect the role of such processes in our present 2D experiments with nonselective microwave pulses that irradiate the whole ESR spectrum.<sup>41</sup>

At room temperature, where thermally activated processes become more important, we may expect a second class of processes to contribute to the observed broadening of the 2D-ELDOR peaks with respect to  $T$ . In particular, phonon modes may be expected to modulate the amplitude and orientation of the malonic acid proton hf and  $g$  tensors<sup>42</sup> which dominate the spin Hamiltonian. This would lead to the relaxation effects outlined in the appendices. However, those phonon modes that are at the low frequency end of the phonon spectrum could induce slow fluctuations in the hf and  $g$  tensors, which can lead to spectral diffusion that broadens the 2D-ELDOR main peaks as a function of mixing time  $T$ . Let us, for the moment, suppose that the dominant phonon modulation is orientational, i.e., there is a slow phonon-induced torsional oscillation in the crystal. We find from Fig. 7(a) that for ( $\theta=30^\circ$ ,  $\phi=0^\circ$ ), a change of 0.3 MHz in resonant frequency  $\omega_+$  would correspond to an angular change of about 1.3°, consistent with a small oscillation. (We get a 1.2° change for  $\theta=50^\circ$ ,  $\phi=0^\circ$ .)

We expect that further studies, especially as a function of temperature, would be helpful to clarify the source of the 2D-ELDOR “motional” cross peaks. We do, however, wish to note that in a preliminary 2D-ELDOR study of a nitroxide spin-labeled organic polymer in a polycrystalline state at room temperature, *no change* was observed in the 2D-ELDOR peaks with mixing time  $T$ .<sup>43</sup> The mechanism



of spin diffusion of the matrix protons should be quite general (except perhaps to the extent to which it is motionally narrowed), whereas phonon modulation should be specific to the medium. Clearly, also the larger concentrations of spins in the malonic acid case could imply contributions from electron-spin-spin interactions mentioned above.

In either case, by observing the real time evolution in a 2D-ELDOR experiment, we observe how each malonic acid radical samples the range of local sites available to it as a result of the slow dynamics. We further note that, whereas the measured  $T_1$ 's and  $T_2$ 's reflect the effects of crystal dynamic modes that are usually fast on the ESR time scale, the 2D-ELDOR "motional" cross peaks are manifestations of the processes that are slow on the ESR time scale.

## V. CONCLUSIONS

The techniques of SECSY ESR, stimulated SECSY, and 2D ELDOR have been applied successfully to a single crystal of irradiated malonic acid and generally excellent agreement with theory is obtained for the nuclear modulation patterns in all these cases.

It has been possible to combine the well-known theories for nuclear modulation in spin-echo spectroscopy and spin relaxation in a single general format in Liouville space. This analysis has confirmed the relative simplicity of two-pulse nuclear modulation experiments with regard to relaxation, viz. only  $T_{2e}$  is of significance. (For simplicity, in our present analysis we ignored any differences of  $T_{2e}$  for the different allowed and forbidden transitions, although such differences are included in the general theory.) However, the analysis explicitly displays the much more complex dependencies on relaxation for the three-pulse nuclear modulation experiments, wherein  $T_{1e}$ ,  $T_{1n}$ ,  $T_{2n}$ , and cross relaxation also play an important role, such that the main allowed peaks and two different classes of modulation peaks are affected differently. It was illustrated how this richness of relaxation behavior could be utilized to obtain all these various relaxation rates from a set of 2D-ELDOR experiments vs mixing time (which can be complemented by stimulated-SECSY experiments). Given the very simple dependence on relaxation of SECSY ESR, this simple two-pulse 2D-FT method would be the easiest to implement in a quantitative fashion for the study of structure by nuclear modulation. The enhanced resolution for structural studies obtained by SECSY ESR was previously demonstrated in frozen samples containing many weakly coupled protons and/or deuterons.<sup>26</sup> The present study has shown that by using very short  $\pi/2$  pulses (5 ns), in conjunction with sufficiently wide bandwidth, it is possible to obtain quantitatively the nuclear modulation patterns from a strongly coupled proton.

The three-pulse 2D-ELDOR experiment leads to a modulation pattern that is very similar to that of SECSY ESR; in fact, they become equivalent for zero mixing time. Thus, whereas the characteristic suppression effect of three-pulse ESEEM is retained in stimulated SECSY, it is

absent in 2D ELDOR. We have shown how by studies as a function of mixing time, exchange peaks can be discriminated from modulation peaks, although there can be oscillations in intensities of some of the modulation peaks as a function of mixing time.

Despite the fact that stimulated SECSY suffers from the suppression effect, it does yield the simplest modulation patterns of the three experiments considered in this work. This can be a virtue in studies involving many nuclei. Intensities of the main allowed peaks and the nuclear modulation peaks do depend somewhat differently on the relaxation, which shows up as homogeneous " $T_2$ -like" effects with respect to  $\omega_T$  (i.e., the mixing time dimension). It should be noted that the relative simplicity of the stimulated SECSY pattern is due in large part to the degeneracy of one class of modulation peaks with the main allowed peaks.

An interesting and potentially important observation was made in the 2D-ELDOR experiment that is characterized as the real time evolution of "motional" cross peaks. This feature should be studied in greater detail as it would allow for the direct study of dynamical processes in solids that are slow on the ESR time scale such as spin diffusion and/or slow phonon modes.

Both 2D-ELDOR and stimulated SECSY are 2D projections of a basically 3D experiment with 2D-ESEEM of Merks and de Beer the other possibility, which, however, lacks the multiplexing advantage. We would suggest that a full 3D experiment be performed by performing 2D ELDOR over a range of mixing times. Afterward, various one- and two-dimensional projections (or slices) of the full 3D experiment could be obtained as desired.

Finally, in this context, we wish to note that the basic premise of this work, viz. to employ full collection of the echo decay resulting from nonselective irradiation of the full spectrum in a FT format, can be combined with recently suggested four- and five-pulse sequences for the study of ESEEM,<sup>44,45</sup> thereby enhancing these pulse sequences by the multiplexing advantage of data acquisition and by the additional spectral dimension.

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## APPENDIX A: THE DENSITY MATRIX ANALYSIS FOR NUCLEAR MODULATION INCLUDING DETAILED SPIN RELAXATION

In this appendix, we first review the basic solutions for nuclear modulation of an  $S=1/2$ ,  $I=1/2$  spin system for two- and three-pulse 2D-ESR. This is carried out in the

usual way in Hilbert space. We then develop the relaxation matrix solution in Liouville space utilizing these results.

### 1. Transformation from the $|S_z I_z\rangle$ basis to the basis that diagonalizes the Hamiltonian in Hilbert space

[This subsection summarizes well-known results (cf. Ref. 4) in order to provide needed definitions of terms.]

$$\mathbf{H} = \begin{matrix} & |1\rangle = |++\rangle & |2\rangle = |+-\rangle & |3\rangle = |-\rangle & |4\rangle = |--\rangle \\ \begin{pmatrix} (C - \omega_n + A/2)/2 & B/4 & 0 & 0 \\ B^*/4 & (C + \omega_n - A/2)/2 & 0 & 0 \\ 0 & 0 & (-C - \omega_n - A/2)/2 & -B/4 \\ 0 & 0 & -B^*/4 & (-C + \omega_n + A/2)/2 \end{pmatrix} \end{matrix} \quad (\text{A2})$$

Note that by Eqs. (4) in the main text,  $A$ ,  $B$ , and  $C$  are functions of the orientational angles  $\beta$  and  $\gamma$ . This Hamiltonian is diagonalized by the similarity transform  $\mathbf{T}^\dagger \hat{\mathbf{H}} \mathbf{T} = \mathbf{E}$ , where

$$\mathbf{T} = \begin{pmatrix} \mathbf{T}_\alpha & 0 \\ 0 & \mathbf{T}_\beta \end{pmatrix} = \begin{pmatrix} c_1 & c_2 & 0 & 0 \\ -c_2 & c_1 & 0 & 0 \\ 0 & 0 & c_3 & c_4 \\ 0 & 0 & -c_4 & c_3 \end{pmatrix},$$

$$\mathbf{E} = \begin{pmatrix} |a\rangle & |b\rangle & |c\rangle & |d\rangle \\ \begin{pmatrix} C/2 + \omega_\alpha & 0 & 0 & 0 \\ 0 & C/2 - \omega_\alpha & 0 & 0 \\ 0 & 0 & -C/2 + \omega_\beta & 0 \\ 0 & 0 & 0 & -C/2 - \omega_\beta \end{pmatrix} \end{pmatrix} \quad (\text{A3})$$

From Eq. (A3), we see that the eigenvectors  $|a\rangle$  and  $|b\rangle$  are linear combinations of  $|1\rangle$  and  $|2\rangle$ , while  $|c\rangle$  and  $|d\rangle$  are linear combinations of  $|3\rangle$  and  $|4\rangle$ . We shall refer to the eigenvectors  $|a\rangle$ ,  $|b\rangle$ ,  $|c\rangle$ , and  $|d\rangle$  as the  $\mathcal{H}_0$  basis. Also,  $\omega_\alpha$ ,  $\omega_\beta$  and the  $c_i$ 's are defined as follows:

$$\omega_\alpha = \left[ \left( \frac{A}{2} - \omega_n \right)^2 + \left| \frac{B}{2} \right|^2 \right]^{1/2},$$

$$\omega_\beta = \left[ \left( \frac{A}{2} + \omega_n \right)^2 + \left| \frac{B}{2} \right|^2 \right]^{1/2}, \quad (\text{A4})$$

$$c_1 = \frac{1}{\sqrt{2}} \left[ 1 \pm \frac{(A/2) - \omega_n}{\omega_\alpha} \right]^{1/2},$$

$$c_2 = -\frac{1}{\sqrt{2}} \left[ 1 \mp \frac{(A/2) - \omega_n}{\omega_\alpha} \right]^{1/2},$$

$$c_3 = \frac{1}{\sqrt{2}} \left[ 1 + \frac{(A/2) + \omega_n}{\omega_\beta} \right]^{1/2},$$

The Hamiltonian for the nuclear modulation problem in the absence of the radiation field is

$$\hat{H} = CS_z - \omega_n I_z + AS_z I_z + \frac{1}{2} BS_z I_+ + \frac{1}{2} B^* S_z I_- \quad (\text{A1})$$

It can be written in matrix form as

$$c_4 = -\frac{1}{\sqrt{2}} \left[ 1 - \frac{(A/2) + \omega_n}{\omega_\beta} \right]^{1/2} \quad (\text{A5})$$

The upper and lower signs in the definition of  $c_1$  and  $c_2$  correspond to the case of  $A/2 > |\omega_n|$  and  $A/2 < |\omega_n|$ , respectively. Clearly  $\omega_\alpha$ ,  $\omega_\beta$ , and the  $c_i$ 's depend on the molecular orientation, i.e., the  $\mathcal{H}_0$  basis is orientation dependent, whereas the  $|S_z I_z\rangle$  basis is not.

#### a. Effect of the pulse

Assuming a short and intense pulse, we can ignore the effects of the spin Hamiltonian  $\hat{H}$  and of the relaxation during the pulse. During the pulse, the density matrix evolves according to

$$\frac{d\rho}{dt} = -i[\hat{\epsilon}(t), \rho], \quad (\text{A6})$$

where  $\hat{\epsilon}(t)$  is the irradiating microwave pulse with intensity  $B_1 = \omega_1/\gamma_e$  and phase  $\phi$ . We have

$$\hat{\epsilon}(t) = \gamma_e B_1 (S_x \cos \phi + S_y \sin \phi) = \frac{\omega_1}{2} (e^{-i\phi} S_+ + e^{i\phi} S_-) \quad (\text{A7})$$

The formal solution can be written for a pulse of duration  $t_p$  as

$$\rho(t_0 + t_p) = \exp(-i\hat{\epsilon}t_p) \rho(t_0) \exp(i\hat{\epsilon}t_p), \quad (\text{A8})$$

where the exponential operator may be written as

$$\exp(i\hat{\epsilon}t_p) = \begin{bmatrix} \cos(\theta/2)\mathbf{1} & i \sin(\theta/2)e^{-i\phi}\mathbf{1} \\ i \sin(\theta/2)e^{i\phi}\mathbf{1} & \cos(\theta/2)\mathbf{1} \end{bmatrix}$$

in the  $|S_z I_z\rangle$  basis. (A9)

Here  $\mathbf{1}$  is the  $2 \times 2$  unit matrix and  $\theta = \omega_1 t_p$ .

Converting this matrix representation of the exponential operator into the  $\mathcal{H}_0$  basis, we have

$$\exp(i\hat{e}t_p) = \begin{pmatrix} \cos(\theta/2)\mathbf{1} & i \sin(\theta/2)e^{-i\phi}\mathbf{M} \\ i \sin(\theta/2)e^{i\phi}\mathbf{M}^\dagger & \cos(\theta/2)\mathbf{1} \end{pmatrix}$$

in the  $\mathcal{H}_0$  basis, (A10)

where

$$\mathbf{M} = \mathbf{T}_\alpha^\dagger \mathbf{T}_\beta = \begin{pmatrix} m_1 & m_2 \\ -m_2 & m_1 \end{pmatrix} \equiv \begin{pmatrix} c_1c_3 + c_2c_4 & c_1c_4 - c_2c_3 \\ c_2c_3 - c_1c_4 & c_1c_3 + c_2c_4 \end{pmatrix}. \tag{A11}$$

Since the pulse propagator depends simply on the  $m_i$ 's, the final signal observed for any arbitrary pulse sequence can be expressed in terms of the  $m_i$ 's. It is therefore convenient to express the  $m_i$ 's and the associated  $k$ 's by some useful relations in terms of parameters in the spin Hamiltonian. They are given in Table III. Note that the case of  $A/2 > \omega_n$  in Table III can be obtained from the case of  $A/2 < \omega_n$  just by interchanging  $\omega_+$ ,  $\omega_-$  and changing the sign of  $\omega_\alpha$ .

**2. The density matrix in the basis that diagonalizes the Hamiltonian in Liouville space (Ref. 46)**

The evolution of the density matrix  $\rho$  is governed by the Stochastic-Liouville equation (SLE)<sup>33</sup>

$$\frac{d}{dt} \rho(\Omega, t) = -i[\hat{H}, \rho(\Omega, t)] - \tilde{\Gamma}[\rho(\Omega, t) - \rho_0(\Omega)], \tag{A12}$$

$\hat{H}$  is the spin Hamiltonian operator, and  $\tilde{\Gamma}$  is a relaxation superoperator. Throughout the paper, “ $\hat{\phantom{x}}$ ” and “ $\tilde{\phantom{x}}$ ” will be used to denote the operator and the superoperator, respectively. The bold character will represent the matrix for the relevant operator. The difference between the time-dependent density matrix and the equilibrium density matrix  $\rho_0$  is denoted as  $\chi(t) \equiv \rho(t) - \rho_0$ , and enables us to rewrite Eq. (A12) as

TABLE III. Relevant coefficients for nuclear modulation.

Constants	$ A/2  \geq  \omega_n $
$k = 4m_1^2m_2^2 = \left(\frac{\omega_n B }{\omega_\alpha\omega_\beta}\right)^2$	The same
$k_+ = m_1^4 = \frac{1}{2} \left(1 - \frac{k}{2} + \sqrt{1-k}\right)$	$\left(\frac{\omega_n^2 - \omega_\pm^2}{\omega_\alpha\omega_\beta}\right)^2$
$k_- = m_2^4 = \frac{1}{2} \left(1 - \frac{k}{2} - \sqrt{1-k}\right)$	$\left(\frac{\omega_n^2 - \omega_\mp^2}{\omega_\alpha\omega_\beta}\right)^2$
$m_1^2$	$\frac{1}{2} \left(1 \mp \frac{\omega_n^2 - (A/2)^2 -  B/2 ^2}{\omega_\alpha\omega_\beta}\right)$
$m_2^2$	$\frac{1}{2} \left(1 \pm \frac{\omega_n^2 - (A/2)^2 -  B/2 ^2}{\omega_\alpha\omega_\beta}\right)$

where  $\omega_\pm = \frac{1}{2}(\omega_\alpha \pm \omega_\beta)$

$$\frac{d}{dt} \chi = -i\tilde{H}\chi(t) - \tilde{\Gamma}\chi(t), \tag{A13}$$

since  $\rho_0 = [\exp(-\hat{H}/kT)] / \{\text{Tr}[\exp(-\hat{H}/kT)]\}$  and thus commutes with  $\hat{H}$ . This can be rewritten in matrix notation in Liouville space as follows:

$$\frac{d}{dt} \mathbf{X} = -i\tilde{H}\mathbf{X} - \tilde{\Gamma}\mathbf{X}, \tag{A14}$$

where  $\mathbf{X}$  is a vector of dimension 16 whose elements are just the 16 density matrix elements.

The Hamiltonian commutator superoperator  $\tilde{H}$  in Liouville space corresponding to the  $|S_z I_z\rangle$  basis can be shown to be block diagonal in four blocks

$$\tilde{H} = \text{Diag}\{\tilde{H}_1, \tilde{H}_2, \tilde{H}_3, \tilde{H}_4\},$$

$$\tilde{H}_1 = \begin{pmatrix} \chi_{12} & \chi_{21} & \chi_{11} & \chi_{22} \\ H_{11}-H_{22} & 0 & -H_{12} & H_{12} \\ 0 & H_{22}-H_{11} & H_{21} & -H_{21} \\ -H_{21} & H_{12} & 0 & 0 \\ H_{21} & -H_{12} & 0 & 0 \end{pmatrix}, \quad \tilde{H}_2 = \begin{pmatrix} \chi_{33} & \chi_{44} & \chi_{34} & \chi_{43} \\ 0 & 0 & -H_{43} & H_{34} \\ 0 & 0 & H_{43} & -H_{34} \\ -H_{34} & H_{34} & H_{33}-H_{44} & 0 \\ H_{43} & -H_{43} & 0 & H_{44}-H_{33} \end{pmatrix}, \tag{A15}$$

$$\tilde{H}_3 = \begin{pmatrix} \chi_{13} & \chi_{14} & \chi_{23} & \chi_{24} \\ H_{11}-H_{33} & -H_{43} & H_{12} & 0 \\ -H_{34} & H_{11}-H_{44} & 0 & H_{12} \\ H_{12} & 0 & H_{22}-H_{33} & -H_{43} \\ 0 & H_{21} & -H_{34} & H_{22}-H_{44} \end{pmatrix}, \quad \tilde{H}_4 = \begin{pmatrix} \chi_{31} & \chi_{41} & \chi_{32} & \chi_{42} \\ H_{33}-H_{11} & H_{34} & -H_{21} & 0 \\ H_{43} & H_{44}-H_{11} & 0 & -H_{21} \\ -H_{12} & 0 & H_{33}-H_{22} & H_{34} \\ 0 & -H_{12} & H_{43} & H_{44}-H_{22} \end{pmatrix},$$

where the matrix elements  $H_{ij}$  are given in Eq. (A2).  $\tilde{H}_1$  and  $\tilde{H}_2$  are in the space spanned by the diagonal and pseudodiagonal elements of  $\chi$  for  $M_s = +1/2$  and  $-1/2$ , respectively.  $\tilde{H}_3$  and  $\tilde{H}_4$  are in the space spanned by the off-diagonal elements of  $\chi$  corresponding to the allowed and forbidden ESR transitions. Note that  $\tilde{H}_4 = -\tilde{H}_3^{\text{tr}}$ . This Hamiltonian superoperator  $\tilde{H}$  is diagonalized by the unitary matrix  $U$ ,<sup>47</sup>

$$\mathbf{U}^\dagger \tilde{\mathbf{H}} \mathbf{U} = \tilde{\mathbf{F}}, \quad \mathbf{U}_{\mu\nu, mn} = \mathbf{T}_{\mu m} \mathbf{T}_{n\nu}^\dagger \quad (\text{A16})$$

$\tilde{\mathbf{F}}$  is the matrix representation of  $\tilde{H}$  in the  $\mathcal{H}_0$  basis. Therefore it is diagonal and each nonzero element is a transition frequency. The unitary transforms  $\mathbf{U}$  and  $\mathbf{T}$  are related by Eq. (A16). The matrix elements for  $\tilde{\mathbf{F}}$  and  $\mathbf{U}$  are given in Eqs. (A17) and (A18)

$$\tilde{\mathbf{F}} = \text{Diag}\{\omega_{\alpha}, -\omega_{\alpha}, 0, 0, 0, 0, -\omega_{\beta}, \omega_{\beta}\}$$

$$C + \omega_+, C + \omega_-, C - \omega_-, C - \omega_+; \quad (\text{A17})$$

$$-C - \omega_+, -C - \omega_-, -C + \omega_-, -C + \omega_+$$

in the  $\mathcal{H}_0$  basis consisting of

$$\{\chi_{ab}, \chi_{ba}, \chi_{aa}, \chi_{bb}, \chi_{cc}, \chi_{dd}, \chi_{cd}, \chi_{dc}, \chi_{ac}, \chi_{ad}, \chi_{bc}, \chi_{cb}, \chi_{ca}, \chi_{da}, \chi_{cb}, \chi_{db}\},$$

$$\mathbf{U} = \text{Diag}\{\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3, \mathbf{U}_3\}, \quad \mathbf{U}_1 = \begin{pmatrix} \chi_{12} & \chi_{21} & \chi_{11} & \chi_{22} \\ c_1^2 & -c_2^2 & -c_1 c_2 & c_1 c_2 \\ -c_2^2 & c_1^2 & -c_1 c_2 & c_1 c_2 \\ c_1 c_2 & c_1 c_2 & c_1^2 & c_2^2 \\ -c_1 c_2 & -c_1 c_2 & c_2^2 & c_1^2 \end{pmatrix} \begin{matrix} \chi_{ab} \\ \chi_{ba} \\ \chi_{aa} \\ \chi_{bb} \end{matrix}, \quad (\text{A18})$$

$$\mathbf{U}_2 = \begin{pmatrix} \chi_{33} & \chi_{44} & \chi_{34} & \chi_{43} \\ c_3^2 & c_4^2 & c_3 c_4 & c_3 c_4 \\ c_4^2 & c_3^2 & -c_3 c_4 & -c_3 c_4 \\ -c_3 c_4 & c_3 c_4 & c_3^2 & -c_4^2 \\ -c_3 c_4 & c_3 c_4 & -c_4^2 & c_3^2 \end{pmatrix} \begin{matrix} \chi_{cc} \\ \chi_{dd} \\ \chi_{cd} \\ \chi_{dc} \end{matrix}, \quad \mathbf{U}_3 = \begin{pmatrix} \chi_{13} & \chi_{14} & \chi_{23} & \chi_{24} \\ c_1 c_3 & c_1 c_4 & c_2 c_3 & c_2 c_4 \\ -c_1 c_4 & c_1 c_3 & -c_2 c_4 & c_2 c_3 \\ -c_2 c_3 & -c_2 c_4 & c_1 c_3 & c_1 c_4 \\ c_2 c_4 & -c_2 c_3 & -c_1 c_4 & c_1 c_3 \end{pmatrix} \begin{matrix} \chi_{ac} \\ \chi_{ad} \\ \chi_{bc} \\ \chi_{bd} \end{matrix}.$$

In Eq. (A18), we indicate the basis of  $U$  in terms of the  $|S_z I_z\rangle$  and the  $\mathcal{H}_0$  basis vectors.

### 3. Spin relaxation

The relaxation superoperator  $\tilde{\Gamma}$  will, in general, display orientation dependence. In problems which involve orientational motions,  $\tilde{\Gamma}$  becomes a rotational diffusion operator and it does not commute with the orientation-dependent terms in  $\tilde{H}$ . For the present experiments on malonic acid crystals, we assume that phonon modulation of hf and g tensors leads to spin relaxation that can be described by the Redfield equation applicable for motional narrowing [i.e., these are the fast phonon modes; slow modes would have different effects (cf. Sec. IV D)]. Then Eq. (A1) for  $\tilde{H}$  is the thermal average over these fluctuations, and the Redfield relaxation matrix  $\mathbf{R}$  is composed from the small fluctuations in these terms, which are summarized in Table IV. Note that in this approach, the density matrix for each orientation of the molecular magnetic tensors relative to the lab frame specified by  $\Omega = (0, \beta, \gamma)$  [cf. Eq. (2)] is taken as uncoupled from all others (this simplification would not be strictly correct if for example there were torsional oscillations). Then the Redfield relaxation matrix  $\mathbf{R}$  will depend on the orientation of the single crystal. Thus we write a separate  $\mathbf{R}$  for each value of  $\Omega$  in the corresponding  $\mathcal{H}_0$  basis. (This is a local approach as opposed to the global approach utilized in motional narrowing problems in which  $\mathbf{R}$  is written in the  $|S_z I_z\rangle$  basis.) Thus we formulate the effects of spin relaxation phenomenologically by the Redfield equation for arbitrary orientation  $\Omega$ ,<sup>48</sup>

$$\frac{d\chi_{\alpha\alpha'}}{dt} = -i\omega_{\alpha\alpha'}\chi_{\alpha\alpha'} + \sum_{\beta, \beta'} R_{\alpha\alpha', \beta\beta'} \chi_{\beta\beta'}, \quad (\text{A19})$$

where  $\alpha, \alpha', \beta, \beta'$  are the spin quantum numbers that represent the eigenstates of the Hamiltonian  $\hat{H}$  expressed by Eq. (A1). At this stage, one has the choice of calculating the full relaxation matrix in the  $|S_z I_z\rangle$  basis (which is relatively easy) and then transform to the  $\mathcal{H}_0$  basis using the transformation defined by Eq. (A18), or alternatively to evaluate the relaxation matrix directly in the  $\mathcal{H}_0$  basis, which is somewhat more difficult. The latter choice of basis does have the virtue that the only important contributions come from the terms that satisfy  $\omega_{\alpha\alpha'} \approx \omega_{\beta\beta'}$ . This simplification does not apply in the  $|S_z I_z\rangle$  basis prior to its transformation. Note that all terms in Eq. (A19) are functions of  $\Omega$ , although we do not display this dependence for simplicity in presentation. In this Liouville space spanned by the  $\mathcal{H}_0$  basis set, the diagonal and the off-diagonal parts of the relaxation matrix are

$$\begin{aligned} \tilde{R}_{\alpha\beta, \alpha\beta} &= -(1/T_2)_{\alpha\beta}, \quad \text{off diagonal,} \\ \tilde{R}_{\alpha\alpha, \beta\beta} &= W_{\beta\alpha}, \quad \tilde{R}_{\beta\beta, \alpha\alpha} = W_{\alpha\beta}, \quad \text{diagonal,} \\ \tilde{R}_{\alpha\alpha, \alpha\alpha} &= - \sum_{\gamma \neq \alpha} W_{\alpha\gamma}, \quad \text{diagonal.} \end{aligned} \quad (\text{A20})$$

In the above discussion, we have emphasized the role of modulation of the dominant interactions in the spin Hamiltonian. In an actual malonic acid crystal, there are also matrix protons which interact weakly by dipolar interactions with the unpaired electrons of the malonic acid

TABLE IV. Illustration of the effect of fluctuation of the various terms in the spin Hamiltonian on the relaxation rates.

	$\mathcal{F}^{a,b}$	$\mathcal{A}^{c,d}$	$ \mathcal{A}_{ac} ^2$	$ \mathcal{A}_{bd} ^2$	$ \mathcal{A}_{ad} ^2$	$ \mathcal{A}_{bc} ^2$	$ \mathcal{A}_{ab} ^{2,e}$	$ \mathcal{A}_{aa} ^2,  \mathcal{A}_{bb} ^{2,e}$
Isotropic $g$	$\frac{\beta_e B_0}{\hbar} \delta g$	$S_z$						$\frac{1}{4}$
Anisotropic $g$	$\delta F \frac{1}{2} (3 \cos^2 \beta - 1) + \delta F^{(2)} \sin^2 \beta \cos(2\gamma)$	$S_z$						$\frac{1}{4}$
	$\delta F \frac{3}{8} \sin(2\beta) - \delta F^{(2)} \frac{1}{4} \sin(2\beta) \cos(2\gamma)$	$S_+ + S_-$	$m_1^2$	$m_1^2$	$m_2^2$	$m_2^2$		$\frac{1}{4}$
Isotropic hf	$-\frac{\gamma_e}{\hbar} \delta a$	$S_z I_z$					$\frac{1}{4} \{\text{Re}(c_1^* c_2)\}^2$	$\frac{1}{16} ( c_1 ^2 -  c_2 ^2)$
		$\frac{1}{2} (S_+ I_- + S_- I_+)$	$\frac{1}{4}  c_2^* c_3 ^2$	$\frac{1}{4}  c_1^* c_4 ^2$	$\frac{1}{4}  c_2^* c_4 ^2$	$\frac{1}{4}  c_1^* c_3 ^2$		
Anisotropic hf	$-\frac{\gamma_e}{\hbar} \left[ \delta D \frac{1}{2} (3 \cos^2 \beta - 1) + \delta D^{(2)} \sin^2 \beta \cos(2\gamma) \right]$	$S_z I_z$					$\frac{1}{4} \{\text{Re}(c_1^* c_2)\}^2$	$\frac{1}{16} ( c_1 ^2 -  c_2 ^2)$
		$-\frac{1}{4} (S_+ I_- + S_- I_+)$	$\frac{1}{16}  c_2^* c_3 ^2$	$\frac{1}{16}  c_1^* c_4 ^2$	$\frac{1}{16}  c_2^* c_4 ^2$	$\frac{1}{16}  c_1^* c_3 ^2$		
	$-\frac{\gamma_e}{\hbar} \left[ \delta D \frac{3}{8} \sin(2\beta) - \delta D^{(2)} \frac{1}{4} \sin(2\beta) \cos(2\gamma) \right]$	$S_z I_+ + S_z I_-$					$\frac{1}{4} ( c_1 ^2 -  c_2 ^2)^2$	$\{\text{Re}(c_1^* c_2)\}^2$
		$S_+ I_z + S_- I_z$	$\frac{1}{4}  c_1^* c_3 - c_2^* c_4 ^2$	$\frac{1}{4}  c_1^* c_3 + c_2^* c_4 ^2$	$\frac{1}{4}  c_1^* c_4 + c_2^* c_3 ^2$	$\frac{1}{4}  c_1^* c_4 + c_2^* c_3 ^2$		
	$-\frac{\gamma_e}{\hbar} \left[ \delta D \frac{3}{8} \sin^2 \beta - \delta D^{(2)} \frac{1}{4} (1 + \cos^2 \beta) \cos(2\gamma) \right]$	$S_+ I_+ + S_- I_-$	$ c_2^* c_4 ^2$	$ c_2^* c_3 ^2$	$ c_1^* c_3 ^2$	$ c_2^* c_4 ^2$		

<sup>a</sup>The interaction constants ( $\delta g, \delta a, \delta F, \delta F^{(2)}, \delta D, \delta D^{(2)}$ ) are the time-dependent deviations from the average values [e.g.,  $\delta g = g(t) - \bar{g}$ ].  
<sup>b</sup>For the orientations chosen in the experiment ( $\gamma = 0^\circ$  or  $90^\circ$ ),  $\sin(2\gamma) = 0$  and  $\cos(2\gamma) = \pm 1$ . For an arbitrary angle, those terms with  $F^{(2)}$  or  $D^{(2)}$  and  $S_\pm$  or  $I_\pm$  become a complex number whose imaginary part is proportional to  $\sin(2\gamma)$ . This general case is not shown in the table for brevity.  
<sup>c</sup>The quantities  $m_1$  and  $m_2$  are given in Table III;  $c_i$  are given by Eq. (A5).  
<sup>d</sup>For compactness, we have not explicitly included cross terms between the various types of terms. These can be written down simply in terms of the products  $\mathcal{F}^q \mathcal{F}^r$ ,  $r \neq q$  in Eqs. (B3) and (B4).  $\mathcal{A}_{\alpha\gamma}^q \mathcal{A}_{\alpha\gamma}^r$  may also readily be written down; e.g., the cross term from  $S_+$  due to the anisotropic  $g$  tensor with  $-\frac{1}{4} S_+ I_-$  from the anisotropic hf term is  $2m_1 \text{Re}(\frac{1}{4} c_2^* c_3)$  for  $\mathcal{A}_{ac}^g \mathcal{A}_{ca}^{\text{hf}} + \mathcal{A}_{ac}^{\text{hf}} \mathcal{A}_{ca}^g$ .  
<sup>e</sup> $\mathcal{A}_{cb} \mathcal{A}_{cc}$  and  $\mathcal{A}_{da}$  are obtained by substituting  $c_1 \rightarrow c_3$  and  $c_2 \rightarrow c_4$  in the entries below.

radical (cf. Sec. IV D). These can contribute to  $T_{2e}$  and  $T_{2n}$  by spin diffusion, especially at low temperatures.<sup>38,39</sup>

If the electron-spin concentration is not very dilute, then electron–electron dipolar (EED) interactions can play a role, as also mentioned in Sec. IV D. Here we emphasize the mutual electron-spin flip flops due to the  $S_{1\pm}S_{2\mp}$  terms which can transfer spins between the two proton hf lines of malonic acid.<sup>11,39</sup> The formal properties of this spin interaction are the same as Heisenberg exchange (HE),<sup>11</sup> so we take advantage of this below and nominally consider the HE mechanism, although a rigorous theory would model the magnitudes of the terms based on EED terms. The  $S_{1\pm}S_{2\mp}$  terms, in causing spin transfer between the hf lines, do not conserve energies of the order of  $A$  [cf. Eq. (A1)], so they must be assisted by phonon modulation of this interaction. (Note that phonon modulation of  $S_{1\pm}S_{2z}$  and  $S_{1\pm}S_{2\pm}$  type terms would contribute to  $T_{1e}$  as well as  $W_x$  for certain orientations.) The theory for relaxation from HE<sup>11,36</sup> is most readily applied in the  $|S_z I_z\rangle$  basis, followed by transformation by  $U$  into the  $\mathcal{H}_0$  basis.

In order to simplify the analysis, we shall choose to introduce the following equalities, which, however, are not general as can be seen from Appendix B and illustrated in Table IV. Nevertheless, they capture the essence of the relaxation problem, while permitting tractable analytic solutions to the problem

$$(i) T_{2e} = (T_2)_{ac} = (T_2)_{bd} = (T_2)_{ad} = (T_2)_{bc},$$

$$T_{2n} = (T_2)_{ab} = (T_2)_{cd},$$

$$(ii) W_e = W_{ac} = W_{bd},$$

$$W_n = W_{ab} = W_{cd},$$

$$W = \begin{pmatrix} \chi_{aa} & \chi_{bb} & \chi_{cc} & \chi_{dd} \\ -W_e - W_n - W_x - \omega_{HE}/4 & W_n + \omega_{HE}/4 & W_e + \omega_{HE}/4 & W_x - \omega_{HE}/4 \\ W_n + \omega_{HE}/4 & -W_e - W_n - W_x - \omega_{HE}/4 & W_x - \omega_{HE}/4 & W_e + \omega_{HE}/4 \\ W_e + \omega_{HE}/4 & W_x - \omega_{HE}/4 & -W_e - W_n - W_x - \omega_{HE}/4 & W_n + \omega_{HE}/4 \\ W_x - \omega_{HE}/4 & W_e + \omega_{HE}/4 & W_n + \omega_{HE}/4 & -W_e - W_n - W_x - \omega_{HE}/4 \end{pmatrix}. \quad (A23)$$

Thus, in the  $\mathcal{H}_0$  basis, we may write Eq. (A19) in matrix form as [cf. Eq. (A14)]

$$\frac{dX}{dt} = (-i\tilde{F} + \tilde{R})X. \quad (A24)$$

The final solutions may thus be obtained by diagonalizing  $(-i\tilde{F} + \tilde{R})$ . However, by virtue of Eqs. (A17), (A22), and (A23), only the  $4 \times 4$  subspace of diagonal elements  $\chi_{ii}$  is still nondiagonal. Since  $\tilde{F}$  in this subspace is just the null matrix, it is just necessary to diagonalize  $W$  given by Eq. (A23) to obtain the normal modes of relaxation. That is,

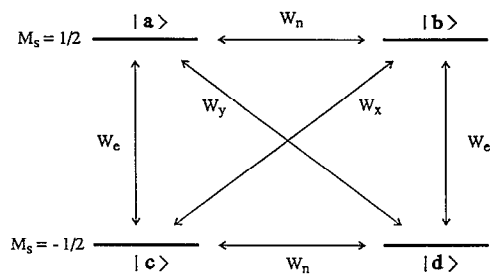


FIG. 16. The relaxation pathways defined in the  $\mathcal{H}_0$  basis

$$W_x = W_{bc} = W_{ad}. \quad (A21)$$

[Considering only  $g$  and hf tensor terms, (i) of Eq. (A21) would be true if  $g$ -tensor contributions dominate  $T_{2e}$  and  $T_{2n}$ ; (ii) of Eq. (A21) also requires the tilt angle  $\beta \cong 0$ , so that  $B \cong 0$ . We can similarly neglect the complication in the treatment of HE requiring its transformation by  $U$ , by either assuming  $B \cong 0$  or else  $|\omega_n| \ll |A/2|$  so  $c_3 \cong c_1$ , and  $c_4 \cong c_2$ . The expressions written below for HE are simplified in this manner.] The relaxation scheme is illustrated in Fig. 16. The relaxation matrix  $\tilde{R}$  in the  $\mathcal{H}_0$  basis becomes:

$$\tilde{R} = \text{Diag} \left\{ -\frac{1}{T_{2n}}, -\frac{1}{T_{2n}}, W, -\frac{1}{T_{2n}}, -\frac{1}{T_{2n}}, \right. \\ \left. -\frac{1}{T_{2e}}, \dots, -\frac{1}{T_{2e}} \right\} \quad (A22)$$

in the same basis as  $\tilde{F}$  [Eq. (A17)]. Note the eigenvalue  $-1/T_{2e}$  is repeated eight times in Eq. (A22).  $W$  is the  $4 \times 4$  matrix corresponding to the diagonal part of  $\chi$ ,

$$S_w^\dagger W S_w = \Lambda_w = \{0, \lambda_4, \lambda_5, \lambda_6\}, \quad (A25)$$

where  $\lambda_4$ ,  $\lambda_5$ , and  $\lambda_6$  are negative real and  $\lambda_3 = 0$ , corresponding to the conservation of total probability. In this normal mode basis,  $(-i\tilde{F} + \tilde{R})$  is a diagonal matrix and can be written as

$$\Lambda = \text{Diag} \left\{ -i\omega_\alpha - \frac{1}{T_{2n}}, i\omega_\alpha - \frac{1}{T_{2n}}, 0, \lambda_4, \lambda_5, \lambda_6, i\omega_\beta - \frac{1}{T_{2n}}, \right. \\ \left. -i\omega_\beta - \frac{1}{T_{2n}}, -i(C + \omega_+) - \frac{1}{T_{2e}}, -i(C + \omega_-) - \frac{1}{T_{2e}} \right\},$$

$$\begin{aligned}
 & -i(C-\omega_-) - \frac{1}{T_{2e}}, -i(C-\omega_+) - \frac{1}{T_{2e}}, \\
 & i(C+\omega_+) - \frac{1}{T_{2e}}, i(C+\omega_-) - \frac{1}{T_{2e}}, i(C-\omega_-) \\
 & \left. - \frac{1}{T_{2e}}, i(C-\omega_+) - \frac{1}{T_{2e}} \right\}. \quad (\text{A26})
 \end{aligned}$$

Note that for the complex eigenvalues corresponding to the various transitions, the real parts are the homogeneous linewidths and the imaginary parts are the transition frequencies. The values of  $\lambda_4, \lambda_5, \lambda_6$  and of  $\mathbf{S}_w$  are quite complicated, in general. However with the assumptions in Eq. (A21), we can get a very simple analytical expression

$$\mathbf{S}_w = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}, \quad (\text{A27})$$

$$\{\lambda_4, \lambda_5, \lambda_6\} = \{-2W_n - 2W_x, -2W_e - 2W_x, -2W_e - 2W_n - \omega_{HE}\}.$$

In summary,  $(-i\tilde{\mathbf{H}} + \tilde{\mathbf{R}})$  expressed most conveniently in the  $|S_z I_z\rangle$  basis is transformed into the normal mode basis by two consecutive unitary transformations  $\mathbf{U}$  followed by  $\mathbf{S}$ ,

$$(\mathbf{US})^\dagger (-i\tilde{\mathbf{H}} + \tilde{\mathbf{R}}) (\mathbf{US}) = \Lambda. \quad (\text{A28})$$

The evolution of the density matrix in the absence of the radiation field then obeys the simple "complex-exponential" form

$$\bar{\mathbf{X}}(t_0+t) = \exp(-\Lambda t) \bar{\mathbf{X}}(t_0), \quad (\text{A29})$$

where  $\bar{\mathbf{X}} = (\mathbf{US})^\dagger \mathbf{X}$ .

#### 4. The pulse propagator in Liouville space

Equation (A8) can be rewritten in Liouville space as follows:

$$\rho_{\alpha\alpha'}(t_0+t_p) = \sum_{\beta, \beta'} \tilde{\mathbf{P}}_{\alpha\alpha', \beta\beta'} \rho_{\beta\beta'} \quad (\text{A30})$$

with the pulse propagator superoperator  $\tilde{\mathbf{P}}$ ,

$$\tilde{\mathbf{P}}_{\alpha\alpha', \beta\beta'} = [\exp(-i\hat{e}t_p)]_{\alpha\beta} [\exp(i\hat{e}t_p)]_{\beta'\alpha'}. \quad (\text{A31})$$

We can rewrite Eq. (A30) in matrix form

$$\mathbf{X}(t_0+t_p) = \tilde{\mathbf{P}}\mathbf{X}(t_0) + (\tilde{\mathbf{P}} - \mathbf{1})\rho_0. \quad (\text{A32})$$

$\tilde{\mathbf{P}}$  given by Eqs. (A9) and (A31) can be expressed most conveniently in the  $|S_z I_z\rangle$  basis in the order shown in Eq. (A15)

$$\begin{aligned}
 \tilde{\mathbf{P}} &= \begin{pmatrix} \cos^2(\theta/2)\mathbf{1} & P_{12} & iP_{13} & iP_{14} \\ P_{12}^\dagger & \cos^2(\theta/2)\mathbf{1} & iP_{23} & iP_{24} \\ iP_{13}^\dagger & iP_{23}^\dagger & \cos^2(\theta/2)\mathbf{1} & P_{34} \\ iP_{14}^\dagger & iP_{24}^\dagger & P_{34}^\dagger & \cos^2(\theta/2)\mathbf{1} \end{pmatrix}, \\
 P_{12} &= \sin^2\left(\frac{\theta}{2}\right) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad P_{13} = \frac{1}{2} \sin \theta e^{i\phi} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
 P_{14} &= -\frac{1}{2} \sin \theta e^{-i\phi} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_{23} = -\frac{1}{2} \sin \theta e^{i\phi} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\
 P_{24} &= \frac{1}{2} \sin \theta e^{-i\phi} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad P_{34} = \sin^2\left(\frac{\theta}{2}\right) e^{-2i\phi} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
 \end{aligned} \quad (\text{A33})$$

Transforming into the  $\mathcal{H}_0$  and normal mode basis, we get

$$\bar{\mathbf{X}}(t_0+t_p) = \bar{\mathbf{P}}\bar{\mathbf{X}}(t_0) + (\bar{\mathbf{P}} - \mathbf{1})\bar{\rho}_0 \quad (\text{A34})$$

with  $\bar{\mathbf{P}} = (\mathbf{US})^\dagger \bar{\mathbf{P}} (\mathbf{US})$  and  $\bar{\rho}_0 = (\mathbf{US})^\dagger \rho_0$ . Using Eqs. (A29) and (A34), the evolution of the density matrix can be calculated for any pulse sequence. Equations (5) and (7) in the main text for SECSY ESR and 2D ELDOR plus stimulated SECSY, respectively, were derived by this procedure.

## APPENDIX B: ORIENTATION-DEPENDENT SPIN RELAXATION

The fluctuating time-dependent perturbation leading to the relaxation can be written in general form as follows<sup>49</sup>:

$$\mathcal{H}_1(t) = \sum_q \mathcal{F}_1^q \mathcal{A}_1^q, \quad (\text{B1})$$

where  $\mathcal{A}_i$  are the spin operators in the lab frame and  $\mathcal{F}_i$  are the time-dependent fluctuating parts of the Hamiltonian, which are defined in the molecular frame and transformed into the lab frame.

For the  $\mathcal{H}_0$  basis, with a significant hf tensor, there are no degenerate energy levels and transitions, so we have only the diagonal elements for the  $T_2$ -type relaxation, i.e.,

$$\begin{aligned} (T_2^{-1})_{\alpha\beta} &= -R_{\alpha\beta, \alpha\beta} \\ &= \int_0^\infty d\tau \overline{\omega_{\alpha\beta}(t)\omega_{\alpha\beta}(t+\tau)} \\ &\quad + \frac{1}{2} \left( \sum_{\gamma \neq \alpha} W_{\alpha\gamma} + \sum_{\gamma \neq \beta} W_{\beta\gamma} \right), \end{aligned} \quad (\text{B2})$$

where  $\omega_{\alpha\beta}(t) = [\mathcal{H}_1(t)_{\alpha\alpha} - \mathcal{H}_1(t)_{\beta\beta}]$ , and the first term on the right-hand side of Eq. (B2) is usually taken as  $T_2'^{-1})_{\alpha\beta}$ .

So

$$\begin{aligned} &\int_0^\infty d\tau \overline{\omega_{\alpha\beta}(t)\omega_{\alpha\beta}(t+\tau)} \\ &= \sum_{q,r} (\mathcal{A}_{\alpha\alpha}^q - \mathcal{A}_{\beta\beta}^q) (\mathcal{A}_{\alpha\alpha}^r - \mathcal{A}_{\beta\beta}^r) \\ &\quad \times \int_{-\infty}^\infty d\tau \overline{\mathcal{F}^q(t)\mathcal{F}^{r*}(t+\tau)} \end{aligned} \quad (\text{B3})$$

and the transition probabilities  $W_{\alpha\gamma}$  which provide  $T_1$ -type relaxation and also contribute to  $T_2$  can be written as

$$\begin{aligned} W_{\alpha\gamma} = R_{\alpha\alpha, \gamma\gamma} &= \sum_{q,r} \mathcal{A}_{\alpha\gamma}^q \mathcal{A}_{\gamma\alpha}^r \int_{-\infty}^\infty \overline{\mathcal{F}^q(t)\mathcal{F}^{r*}(t+\tau)} \\ &\quad \times \exp(-i\omega_{\alpha\gamma}\tau) d\tau. \end{aligned} \quad (\text{B4})$$

The specific role of the  $g$  and  $hf$  tensors is illustrated in Table IV for the  $\mathcal{H}_0$  basis. Note that not only are the  $\mathcal{F}^q$  orientation dependent, but the  $\mathcal{A}_{\alpha\gamma}^q$  can also show substantial orientation dependence. Utilizing Table IV and Eqs. (B2)–(B4), it is readily possible to write down all the relevant (orientation-dependent)  $T_2$ 's and  $W_{\alpha\gamma}$ 's. Whereas

this leads in general to complex expressions not amenable to analytical solution, our general computer program permits the calculation of the 2D-ESR spectrum for any case of relaxation in the high temperature limit.

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