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Studies of Nonlinear Spin Oscillations in a Rarefied Quantum Gas: Spin Polarized Atomic Hydrogen

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We describe pulsed nuclear magnetic resonance experiments probing the evolution of the spin wave frequency spectrum from the linear into nonlinear regimes. Results compare favorably with numerical simulations based on the theoretical equations of motion and indicate the presence of distinct modes not present in the linear regime.

1. INTRODUCTION

Spin transport in spin polarized atomic hydrogen can be described by a set of highly nonlinear local equations of motion which couple transport of all components of magnetization in all spatial directions [1-3]. For small NMR tipping angles the equations can be linearized and the longitudinal and transverse magnetization components become decoupled. In fact, the resulting equation of motion for the transverse spin density is analogous to a Schrödinger equation with damping [1,2,5] or in a limiting nonlinear regime, to a dissipative Heisenberg ferromagnet [4]. The fully nonlinear equations have been studied theoretically by Lévy [4] for the model case of zero static magnetic field gradient and one spatial dimension. To date however, pulsed magnetic resonance experiments on these systems have been focused on the linear response [5]. We present some results on experiments in the fully nonlinear regime, and in particular, on the evolution of the frequency spectrum as the spin system is disturbed increasingly far from equilibrium with large tipping angle NMR pulses.

2. EXPERIMENT

In approaching these experiments we sought to simplify the equations of motion by minimizing gradients in the applied static magnetic field. We expected that this would help to isolate any nonlinear effects by minimizing the coupling to the "linear" spin wave modes, while also providing a basis for a comparison with the predictions of Lévy. The experimental geometry was identical to that used in earlier work [5] and will not be described here.

The static field gradient was adjusted using three linear gradient and one quadratic gradient shim coils. The field was tuned to yield a minimum in the overall spectral line width (full width at half maximum). To avoid ambiguities introduced by the presence of spin waves, the adjustment was made under conditions of low nuclear polarization using small NMR tipping angles. Using this technique we were able to achieve an field gradient whose magnitude was about 0.1 Gauss/cm in a static field of 7.5 Tesla. Based on the final line shape and known field profiles for our magnet, we interpret the residual static field gradient as primarily quadratic in form.

Under these conditions we performed a series of isolated NMR pulses of increasing tipping angle up to 90 degrees. Here the tipping angle in a measure of the pertubation of the system from equilibrium and hence of the relative importance of the nonlinear terms in the transport equations. Sweeps were performed by successively increasing tipping angle from 0 to 90 degrees and then repeating the sequence in reverse order. By sweeping angle in both directions we were able to test for any effects of density changes during the experiment which may result from NMR induced recombination [9]. Sufficient time was allowed between individual pulses for complete recovery of the polarization within the resonator by diffusion of atoms into the resonant cavity [7]. The experiment was performed at fixed temperature, high initial nuclear polarization and constant density ($P_z \approx 1$).

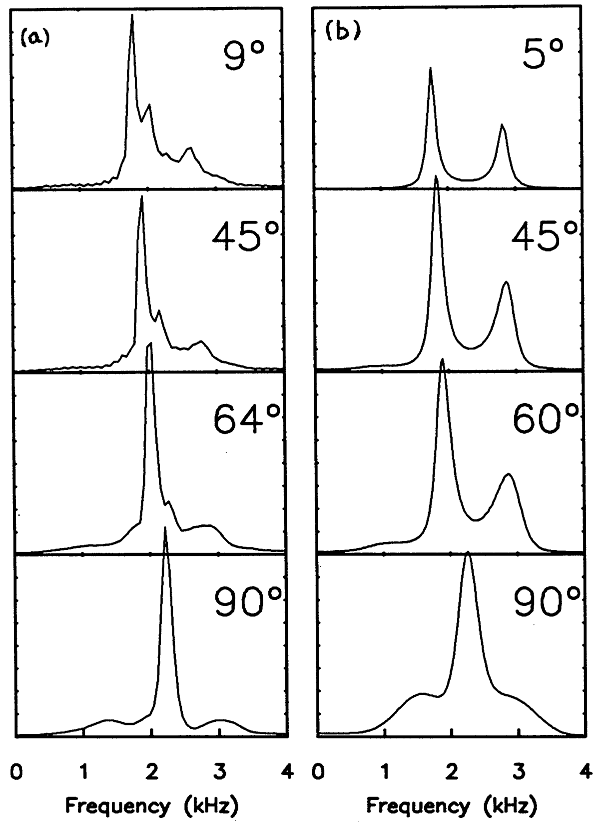

Fig. 1. Pulsed NMR spectrum as function of tipping angle (a) results from experiments performed at T = 300mK, $n \sim 10^{16} cm^{-3}$ and $P_z \sim 1$ (b) results of computer simulations of experiment.

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3. RESULTS

The variations observed in the NMR spectrum are characterized in figure 1a. As the tipping angle increases from the small angle regime (about 5 to 10 degrees) we observe a frequency increase in the peaks present in the initial spectrum, where the rate of increase with tipping angle is greater for the lower frequency peak than for the higher frequency peaks. In addition we observe an increase in the peak width with increasing tipping angle where the rate of increase is larger for the higher frequency peak. Most striking, however, is the appearance of a new peak at a frequency that is *lower* than the lowest frequency mode present in the small angle results.

It may be useful to discuss these results in the language used to describe the linear experiments, in particular using the Schrödinger analogy. For the case where the static field gradient is quadratic in nature, the linear spin wave modes are essentially harmonic oscillator modes. For our resonator we couple only to the symmetric modes [5] and therefore, we can attribute the two prominent peaks observed in the small angle spectra to these lowest two even modes. The small peak on the side of the most prominent peak is thought to be due to a weakly coupled mode in an orthogonal spatial direction. As larger tipping angles are used, the nonlinear terms contribute to a deformation of the linear modes. In this picture, the peak present below the first main peak would correspond to the development of a "new" mode in the "bottom" of the quadratic well. We note that the frequency of the "new" mode does not correspond to any sensible combination of the frequencies of the other modes, suggesting that this peak is truly a new mode rather than a mixing of the other two modes. We have compared our results with the predictions of Lévy and are unable to describe the observed effects in the context of his work.

5. COMPUTER SIMULATION

In an effort to increase our understanding of these effects, we have performed computer simulations similar to those performed by Fermi, Pasta and Ulam [6]. At present, the simulations are restricted to the simplified case of a system of 101 spins evenly spaced in one dimension but otherwise allowed to evolve according to the fully nonlinear transport equations. The static field gradient used in the simulation was chosen to match the experimental conditions. Values chosen for μ, the spin rotation quality factor, and D_o, the self diffusion coefficient, were based on independent measurement [7]. The results of the simulation are depicted in figure 1b. The qualitative behavior of the results are identical to those seen in the experiment, both for the behavior of the modes present in the linear regime and in the occurrence of the new peak at large angles.

The use of the simulation allowed us to examine the time evolution of both longitudinal and transverse magnetization. The time development of the longitudinal magnetization for the large tipping angle spectra showed a rather striking effect. When a 90 degree pulse is applied uniformly to all of the spins, the longitudinal magnetization is initially zero, as expected. However, as time passes, a region of nonzero z magnetization develops at each boundary, with the sign of the magnetization at one side opposite to that of the other. As time passes this pattern appears to grow outward from the boundaries until a sinusoidal pattern is established across the "cell". At later times, this pattern is observed to smooth out, leaving no net longitudinal magnetization. This result is consistent with our understanding of the spin transport equations and can be attributed to spin rotating collisions which rotate magnetization out of the transverse plane while leaving the net z component of magnetization unchanged. These effects have been discussed earlier in this system in the context of spin echo experiments [8]. We plan to test for the presence of this effect in our experiment by following the initial 90 degree pulse with a second probe pulse and studying the resulting signal.

The behavior of the longitudinal magnetization can be argued as being related to the observed increase in line width of the peaks present in all spectra. Collisions which give rise to a conversion of transverse into longitudinal magnetization can be thought of as providing another channel for the depletion of transverse magnetization and therefore causing an increase in line width of the transverse modes.

6. CONCLUSION

In future experiments we plan to probe in more detail the sensitivity of these effects to field gradients as well as initial polarization and temperature. In addition we have developed linear predictive techniques for analyzing the data which we are using to gain a more quantitative understanding of the behavior of the observed effects.

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